Computers and Chemical Engineering 134 (2020) 106690

Contents lists available at ScienceDirect

Computers and Chemical Engineering

journal homepage: www.elsevier.com/locate/compchemeng

A two-stage procedure for efficiently solving the integrated problem of production, inventory, and distribution of industrial products

Mariana E. Cóccola [a,b], Carlos A. Méndez [b], Rodolfo G. Dondo [b,*]

[a] Universidad Tecnológica Nacional, Facultad Regional Concepción del Uruguay, Ing. Pereyra 676, (3260) Concepción del Uruguay, Entre Ríos, Argentina
[b] INTEC (Universidad Nacional del Litoral - CONICET), Güemes 3450, (3000) Santa Fe, Santa Fe, Argentina

ARTICLE INFO

Article history:
Received 21 August 2019
Revised 18 December 2019
Accepted 21 December 2019
Available online 24 December 2019

Keywords:
Production planning
Inventory routing
Multi-compartment vehicles
Column generation
MILP

ABSTRACT

This paper deals with the problem of optimally planning the production, inventory and distribution of products transported via multi-compartment vehicles. It assumes that facilities in the distribution network have preservation-storing devices to inventory products on-site. Production activities may be performed on any time period of the planning horizon. Due to problem complexity, a two-stage solution strategy that first generates a set of multi-period distribution routes through a column generation approach is proposed. The routes are used for feeding the MILP formulation of the problem. Several valid inequalities are proposed for expediting the MILP resolution. The aim is to maximize the profit obtained by the company that fabricates and distributes the products. This profit is computed as the total income from sales minus the sum of all costs incurred along the planning horizon. The effectiveness of the two-stage solution strategy is tested on an extensive set of realistic instances.

1. Introduction

Due to the increasing pressure for reducing costs, inventories and ecological footprint, and in order to remain competitive in the global marketplace, Enterprise-wide Optimization (EWO) has become a major goal of the chemical industry (Grossmann, 2012). The global scale of chemical and food industries force to better integrate production, inventory and distribution decisions because of fluctuating demands and seasonal imbalances of raw materials and products flows. The consolidation of production, inventory and distribution efforts is a challenging problem for companies trying to optimize their supply chain (Bard and Nananukul, 2010). In the downside of a typical supply chain, each individual process is often planned and optimized using predetermined decisions from upstream activities. For example, a production planner first takes the production lot-sizing decisions in order to minimize production and inventory costs and later such production decisions become the inputs of the distribution planning problem. This sequential approach may greatly reduce the operating margins obtained by the company with regards to an integrated logistics management.

The production routing problem (PRP) is an operational planning application that simultaneously optimizes production, inventory, routing and delivering decisions. In the PRP problem, the planner must decide how much to produce at each time period. The production activities have associated fixed setup costs and variable production costs related to the produced quantity. In addition, the lot size cannot exceed the production capacity. The product deliveries from the plant to retailers are fulfilled by a fleet of capacitated vehicles considering both fixed and variable routing costs. In addition, if products are stored at the plant and retailers, inventory holding costs are incurred. The PRP problem has a practical relevance for the Vendor Managed Inventory (VMI) approach, in which the supplier, acting as the central decision maker, monitors the inventory on retailers in order to plan the replenishment policy (Adulyasak et al., 2015). In a traditional relationship, where customers call their orders, large inefficiencies may occur due to the timing of customers' orders leading to mismatches between products availability and products demand, which force the producer to incur in high inventory and distribution costs. The VMI approach allows smoothing demand variability and reducing inventory holding and distribution costs. Achieving cost savings for VMI partnerships, however, is not an easy task, particularly with a large number and variety of customers (Savelsbergh and Song, 2008). According to such a partnership, customer inventories are replenished by the vendor using monitoring and forecasting in a way that each product-inventory on each customer must be replenished so as to never fall under its safety level. The supplier manages inventory on customers, deciding when and how much to deliver to each one of them. Customers benefit from higher service levels and greater product availability because vendors can use

* Corresponding author.
E-mail address: rdondo@santafe-conicet.gov.ar (R.G. Dondo).

https://doi.org/10.1016/j.compchemeng.2019.106690
0098-1354/

Nomenclature

Subscripts

c	vehicle's compartments
i,i',j	suppliers or customers
k	products
r	routes
t,t'	periods of the planning horizon

Sets

A	minimum-distance arcs interconnecting suppliers and customers
C	vehicle's compartments
I^-	customers
I^+	production plants
Inc_{it}	subset of incompatible customers for customer i during period t
K	products
R	feasible routes
R'	feasible routes generated by the CG approach
Suc_{it}	subset of customers that can be successor of customer i during period t
T	periods of the planning horizon

Binary variables

S_{ij}	variable sequencing locations i and j along a route
X_r	variable determining if route r is selected
Y_i	variable determining that site i belongs to the route designed by the routes-generator problem
Y_{itk}	variable denoting the production of product k on plant i during period t
Z_{it}	variable computing if customer/plant i is visited during period t
W_{kcr}	variable determining if product k is allocated on compartment c of route r

Continuous variables

Λ_{ikrt}	quantity of product k picked(delivered) from(to) site i by route r during period t
$C\_$	cost of route generated by the pricing problem
$D\_$	overall travelled distance
D_i	distance travelled to reach customer i
I_{itk}	inventory of product k stored on plant/customer i during period t
I_{rk}	in-route inventory of product k carried by the vehicle traveling route r
P_{itk}	quantity of product k made on plant i during period t
T_i	time spent to reach customer i
$T\_$	overall traveling time

Parameters

a_{irt}	binary parameter stating that route r visits location i during period t
a_{it}	earliest service time at customer i during period t
b_{it}	latest service time at node i during period t
c_{ik}	unit production cost of product k on plant i
c_r	cost of route r
c_{ik}^{inv}	cost of inventorying a unit of product k on customer (plant) i
$c_k^{in-route}$	in-route inventory cost of product k
c_{ik}^{setup}	setup cost for production of product k on plant i
cf	fixed vehicle utilization cost
cv	travel-time unit-cost
d_{ij}	distance between locations i and j
dem_{itk}	demand of product k by customer i during the period t
i_{ik}^0	initial inventory of product k on location i
i_{ik}^{min}	minimum inventory level or safety-stock for product k on location i
i_{ik}^{max}	maximum storage capacity of product k on location i
M_D, M_T	upper bounds for travelled distance *(D)* and travel time *(T)* variables
p_k	selling price of a unit of product k
p_{ik}^{min}	minimum production capacity of product k at plant i
p_{ik}^{max}	maximum production capacity of product k at plant i
q_c	cargo capacity of compartment c
$speed$	average speed of trucks
st_i	service time on customer (plant) i
t^{max}	maximum vehicle routing time
t^0	earliest start time of the route
t_{ij}	traveling time between locations i and j
$time_t$	end time of period t
v_i	minimum number of visits necessary to satisfy the demand of customer i
v^+	minimum number of necessary routes
vi_r	index value of route r
y_k^{min}	minimum number of startups necessary to satisfy demand of product k
y_k^{max}	maximum number of startups necessary to satisfy demand of product k
σ_i	dual variable associated to constraint (20)
π_{it}	dual variable associated to constraint (21)
β	dual variable associated to constraint (22)

inventory data at customer to predict future demands (Fumero and Vercellis, 1999).

On the other hand, the inventory routing problem with continuous moves (IRPCM) is a problem specifically designed to handle limited product availabilities at production facilities because some customers cannot be served using out-and-back tours. In this case, delivery tours can last for several days (Savelsbergh and Song, 2008). Although the problem of simultaneously planning the production and inventory of multiple products in several plants to later distribute them through multi-period routes can be found in several industrial environments, almost no literature about the subject is available. Most contributions in the literature have been focused on single-period routes and single product variants of the PRP. Consequently, this paper is an attempt to advance the research about the integrated planning of production, inventory and delivery of several products from multiple plants to customers by using routes that may cover more than just a single time-period. The routes are performed by multi-compartments vehicles. In addition, routes start-times may be adjusted to further achieve cost-savings. These are particularly hard features that have not yet been considered in the context of the PRP.

The current work aims at modelling and optimizing the integration of production, inventory and distribution of several products requiring non-negligible preservation costs. It proposes a two-stage procedure that first uses the column generation paradigm (CG) for generating distribution routes, which are then used to feed a MILP formulation of the integrated problem. The proposed solution strategy is able to efficiently solve large size realistic instances. This paper continues a line of research on the optimal integration of production, inventorying and distribution of chemical fluids. Previous works by Cóccola et al. (2017, 2018) focusing respectively on optimizing the order-based-resupply of chemical fluids and on the inventory and routing of several fluids trough a fleet of homogeneous multi-compartment trucks respectively can be found in the literature. The main contributions of this third work are the following:

1. A formulation for modelling the optimal planning of production, inventory and distribution of several industrial products over a multi-period time horizon. The case of bulk delivery by a homogeneous fleet of multi-compartments trucks is here researched.
2. A decomposition strategy based on a column generation algorithm for generating distribution routes to feed to the MILP formulation of the problem. The idea behind this is to decouple delivering decisions from production and inventory decisions with the purpose of tackling industrial size instances in reasonable computation times. Valid inequalities to strength the mathematical formulation are also proposed.
3. Computational experiments on both standard and realistic instances featuring different sizes and characteristics are performed to test the capability of the proposed algorithm for providing effective and efficient solutions.

The remainder of this paper is organized as follows: In Section 2 a literature review is performed focusing on different problems involved by the integrated planning of production, inventory and distribution of several industrial products. Section 3 describes the problem of optimally assembling decisions about production, inventorying and distribution. Section 4 formally states the problem. A MILP formulation of the problem based in the enumeration of feasible routes is presented in Section 5. The proposal of separating routing decisions from production and inventorying decisions is presented in Section 6. The two-stage procedure is explained in Section 7. Numerical tests over a series of instances featuring different sizes and different timing and spatial characteristics are presented in Section 8. Finally, the concluding remarks are stated in Section 9.

2. Literature review

One of the most-referenced logistic problems in the literature is the lot-sizing problem with direct shipment, which assumes that a product is transported directly from the manufacturing plant to the customers. The problem goal is to minimize the overall cost over a given planning horizon. Several researchers have studied this production and distribution problem with direct shipment, considering the distribution cost just as a fixed cost. The review by Aduyasak et al. (2015) surveys main contributions on the subject. When the routing decisions are included but the production activities are disregarded, the problem becomes an inventory routing problem (IRP), which assumes that each plant has an available quantity of the product at beginning of planning horizon to be distributed between the customers. Besides determining the delivery quantities and routes to serve customers, the IRP problem also includes timing decisions in order to determine when to service each customer. The IRP first appeared in a gas delivery study by Bell et al. (1983), which proposed a solution strategy based on Lagrangian relaxation. The IRP is a difficult combinatorial optimization problem, both theoretically and practically, because of the complex periodic routing and the inventory decisions involved in it. Many researchers have tried to solve this logistic problem through different solution strategies. Two comprehensive reviews about the IRP problem and its solution approaches can be found in Andersson et al. (2010) and Coelho et al. (2014). With regard to exact solution strategies, two branch-and-cut procedures were developed by Archetti et al. (2007) and Solyalı and Süral (2011) for solving the IRP problem with a single capacitated vehicle. Savelsbergh and Song (2008) considered the IRP with continuous moves where a product is distributed from a set of plants to a set of customers by multiple vehicles traveling along multi-period routes. More recently, Etebari and Dabiri (2016) proposed a quadratic mixed-integer programming model for the single product, multi-period IRP under the dynamic pricing. These authors developed a simulated annealing framework, which has embedded in it a heuristic approach comprising initialization, demand generation, demand adjustment, inventory routing, and neighborhood search phases. On the other hand, Dong et al (2017) presented several alternative algorithms for solving IRP problem variants. Variants of the problem in a maritime context have been proposed by Christiansen and coworkers. See e.g. Christiansen et al. (2004); Christiansen et al. (2007) and Christiansen et al. (2013). They applied Dantzig–Wolfe decomposition to solve these problems.

Generally, the lot-sizing problem and the IRP problem have been treated separately in the literature, leaving aside the potential benefits provided by an integrated solution strategy. The lot-sizing problem with direct shipment disregards routing decisions, while the IRP ignores production activities. In the last years, the operational research on supply chains has been moving towards the development of integrated approaches in where production, inventory, and routing decisions are taken together. To the best of our knowledge the benefits of coordination between production and routing activities was first researched by Chandra (1993) and Chandra and Fisher (1994). Bard and Nananukul (2010) proposed a heuristic based on a branch-and-price framework to solve a single-plant single-product PRP without continuous moves. Although the algorithm may be used in its exact way, the authors also proposed to use it in a heuristic mode to obtain good solutions in reasonable CPU times. In addition, Archetti et al. (2011) discussed the PRP under the maximum level and order-up-to-level policies and developed a mixed integer programming heuristic to solve the problem. The proposal of these authors was tested on several problem instances considering a simple PRP involving an un-capacitated production plant and constants demands. The distribution problem is solved as a shortest path problem with the goal of determining the best single-period distribution routes. On the other hand, some papers have introduced exact algorithms to compute strong lower bounds in order to find the optimal solution to the PRP problem. For example, Ruokokoski et al. (2010) and Archetti et al. (2011) employed a branch-and-cut approach similar to a previous proposal by Archetti et al. (2007). Adulyasak et al. (2014) focused on the PRP with multiple vehicles and proposed two branch-and-cut approaches based on different formulations to the problem. More recently, Cóccola et al. (2013) proposed a MILP framework for integrating production and transportation activities in multi-echelon supply chains. Even though the proposal of these authors achieves a proper coordination of activities, the management of inventories is disregarded.

In the context of a VMI approach, manufacturers that supply many retailers on a periodic basis must ideally formulate an optimal replenishment strategy. Due to the combinatorial complexity of this aim, several pragmatic heuristic approaches have been developed and used by the practitioners. One approach proposes to use a balanced strategy in which an equal proportion of retailers are replenished each time period of the planning horizon. This strategy has the advantage of balancing the workload at the plant. A second approach called "synchronized replacement" is based on replenishing all retailers and goods are moved into the manufacturer's warehouse immediately prior to distribution (Bard and Nananukul, 2010). This strategy advocated by many practitioners unbalances workloads at the plant but allowing for cross-docking of a significant portion of the goods (Cheung and Zhang, 2008). For just-in-time suppliers, it is common to partition the customers into compact sets and follow the same delivery sequence daily, skipping those locations with absent demand (Çetinkaya et al., 2009). Inspired by industrial contexts where VMI policies are applied, Neves-Moreira et al. (2019) developed an advanced three-phase methodology for solving a rich variant of the PRP involving several realistic features like production of several families of products in alternative production lines, delivery time-windows and a maximum-level inventory policy. These authors assume that a fixed-size fleet of capacitated vehicles is used for distributing the products from a single plant to several customers through one-period routes. Although our proposal partially overlaps it, this paper broadens the reach of the solution approach for solving realistic problems involving several production plants dispersed over a wide geographical area as well a fleet of unspecified number of multi-compartment vehicles and multi-period routes that may cover several days. In reference to the paper presented by Neves-Moreira et al. (2019), it is worth to remark that a complete review about the scarce literature dealing with realistic PRP environments has been developed by the authors.

The VMI methodology has been widely used for planning the production and distribution of industrial fluids in the chemical industry. Industrial fluids like liquid oxygen and liquid nitrogen have useful lives much longer than the planning horizon while some food products like milk and its derivative products have shelf lives in the order of the planning horizon. An overview about contributions on the design of food distribution networks is presented by Akkerman et al. (2010). Liquid products can be stored on-site in storage tanks but gaseous products usually cannot be stored and industrial fluids companies serve customers through three main distribution modes: large process plants, cryogenic liquid and packaged gases (Barbosa-Povoa et al., 2018). For large customers (e.g. refineries, steel mills), industrial fluids plants operate adjacent to their facilities and distribute products by pipelines. Medium-size customers, such as hospitals and universities, typically have liquid storage devices used to replenish the products. The third mode involves packaged fluids. Carrier cylinders used to transport fluids are owned by the company. In addition to cylinders, stores also retail hard goods that are purchased from vendors and shipped through distribution centers. These ones, in turn, either ship products directly to customers or to the stores for pickup or delivery (Barbosa-Povoa et al., 2018). Marchetti et al. (2014) proposed a multi-period mixed-integer linear programming model for the optimal production and distribution of industrial gases. Their objective was to minimize the total cost of producing and distributing the gases by coordinating decisions at multiple plants and depots. The methodology proposed includes a MILP model for planning the production of fluids and a heuristic to design distribution routes. The paper highlights the benefits of an optimal level of coordination. Since the high computational burden of the problem researched by Marchetti et al. (2004) becomes a major limitation when dealing with industrial size instances, more recently, Zamarripa et al (2016) proposed a rolling horizon algorithm with two different aggregation strategies for decomposing the problem into smaller subproblems. The first strategy relies on the linear programming (LP) relaxation for some binary variables, while the second one uses a model tailored for the distribution side-constraints. On the other hand, Singh et al. (2015) considered a multi-period IRP with multiple products assuming deterministic demand-rates and formulated a linear mixed-integer program to model the problem. As solution strategy, the authors proposed an incremental approach based on decomposing the set of customers of the original problem into smaller sub-problems. A sub-problem is incrementally solved by using a randomized local-search heuristic method with the number of customers growing successively by providing the solution of the previously solved sub-problem as an input.

3. Integrating the production, inventory and distribution problems

The integrated production, inventory and distribution problem researched in this paper involves the production and shipping of several products from multiple factories to customers through a homogenous fleet of multi-compartment vehicles. Since such operations are performed on a known network infrastructure, the integrated problem can be defined on the basis of a set of nodes representing facilities placed at fixed locations. Customers are equipped with multi-commodity preservation-and-storage facilities and similarly, each plant has a multi-commodity preservation-and-storage facility from which the products to be fabricated and stored can be loaded on vehicles that later transport them to customers. Also, the routes used for products-shipping may last for several time-periods along the planning horizon. All features of the integrated logistic problem are illustrated in Fig. 1.

Each customer consumes one or several products, which are sourced from plants producing them. It is assumed that forecasted products-demands for each time period of the planning horizon are known data. The supplier is responsible for keeping the inventory level at each facility between the minimum stock level and the maximum stock level at all time periods. The minimum level is the safety stock level, below which the stock of products should not fall. The inventory costs in all sites (plants and customers) are covered by the supplier. In-route inventory-costs are also considered and covered by the supplier. Hence, the following issues must be addressed by the planner:

1. What quantity of products must be produced during each time-period of the planning horizon?
2. When to resupply a given customer?
3. Which clients to deliver at each period and from which plant?
4. What quantity of products must be supplied to each visited customer?
5. How many vehicles must be used?
6. How to fill each vehicle-compartment for servicing their allocated customers?
7. What periods of the planning horizon must a given vehicle trip cover?

Considering the complexity and the dimension of the integrated problem when realistic instances are considered, it is unpractical to solve a monolithic mathematical formulation for finding solutions useful as answers to the above questions. Therefore, this paper proposes a two-stage solution strategy with the aim of finding practical solutions to large instances of the problem.

4. Problem definition

The integrated production, inventory, and distribution problem can be defined on the basis of a set of facilities placed on fixed locations. Such facilities stand for supply plants and customers. The factories, denoted by the subset $I^{+}= \{i_{1+}, i_{2+}, \ldots i_{n+}\}$, produce an unknown quantity P_{itk} of commodity $k \in K$ during each period t of the planning horizon $T = \{t_1, t_2, \ldots, t_t\}$. A homogenous fleet of multi-compartment trucks is utilized for moving the products to customers $I^{-}= \{i_{1-}, i_{2-},\ldots,i_{n-}\}$. It is assumed that every vehicle has $|C|$ compartments of capacity $q_{c.}$ Each facility $i \in I^{-}$ representing a customer is characterized by a known quantity dem_{itk} of commodity $k \in K$ demanded during period t. The vehicles perform load operations at supply plants while unload activities are accomplished at customer locations. Each load/unload operation consumes a fixed time denoted by st_i. For storing the products, both plants $i \in I^{+}$ and customers $i \in I^{-}$ have multi-compartment devices

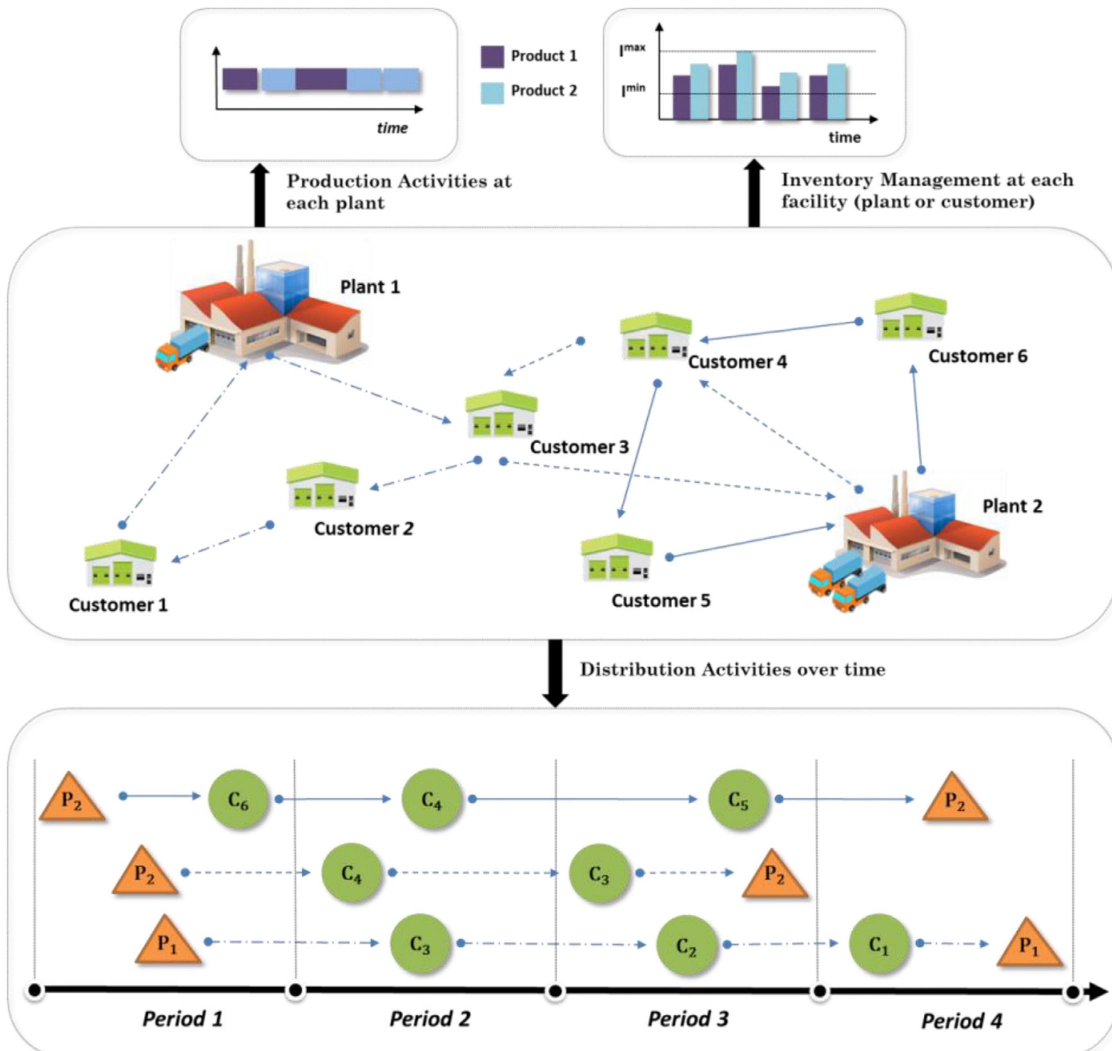

Fig. 1. Overview of the integrated production, inventory, and distribution problem.

with a storing capacity defined by (i_{ik}^{max}-i_{ik}^{min}), where i_{ik}^{max} is the maximum storing capacity and $i_{ik}^{min}$is the minimum operative capacity or "safety stock" below which the inventory of the product must never fall at the end of any time-period. Also, for each product $k \in K$, a known initial inventory i_{ik}^{0} at the start of planning horizon is assumed at both on suppliers $i \in I^{+}$ and customers $i \in I^{-}$. Although perishability is not formally considered in the problem definition, this feature can be implicitly taken into account by the maximum storing capacities, the rotation rates on each client and the product shelf lives. The product rotation rate at each facility must have a value such that the maximum number of days to sell a given unit of product is smaller than the product shelf life. Minimum product rotation-rate is defined in the supplementary information. Specific issues related to food distribution like quality decay and food safety are out of the scope of this work. We refer to the excellent review by Akkerman et al. (2010) that surveys these and other challenges in this specific area.

All facilities integrate a network represented by a graph $G(I^{+} \cup I^{-}, A)$, where A is the set of minimum-distance arcs interconnecting suppliers and customers. Such arcs correspond to road segments characterized by a length d_{ij} and a travel-time t_{ij}, being this last one computed as the distance d_{ij} divided by the average *speed* of in-route trucks. A vehicle route $r \in R$ is considered feasible when the vehicle compartments-capacity constraint is satisfied and the overall travelling time is lower than a maximum time-length t^{max}. Costs incurred for fulfilling customers' demands are:

(i) Production costs comprising both setup costs $c_{ik}^{setup}$for enabling the production of k and variable costs, which are proportional to the produced quantity P_{itk}. The unit production-cost is denoted by c_{ik}.
(ii) Transportation costs for delivering products from plants to customers comprising both vehicles fixed-utilization cost cf and travelling costs proportional to the travelled time. The travelling cost per unit time is denoted by cv.
(iii) Inventory costs associated to the quantity I_{itk} of product k stored at each time period t. The unit holding cost is given through parameter c_{ik}^{inv}.
(iv) In-route inventory costs incurred by the vehicles transporting the products from plants to customers. The cost associated to the in-route maintenance of a unit of product k is denoted by $c_{k}^{in-route}$.

The integrated production, inventory, and distribution problem aims at determining:

(i) The quantity of products fabricated in each plant during every time period of the planning horizon.
(ii) The inventory profiles such that, for each product, the maximum storing capacity is respected and no stock-outs occur both on customers and on plants.

(iii) The quantity of products delivered in each vehicle route to any visited customer.

The objective is the maximization of the company profit defined as the difference between the total income obtained by the sales of products to the customers and the total costs incurred to satisfy customers' demands; i.e. the summation of transportation costs, inventory holding costs (both at suppliers and customers), in-route inventory costs, and total production costs on plants.

5. Mathematical formulation

The production, inventory and distribution problem (PIRP) on a multi-site system defined above can be mathematically represented through a mixed integer-linear problem (MILP). Let R be the set of all feasible multi-period replenishment routes from plants to customers. Each feasible route $r \in R$ is characterized by a cost c_r given by the sum of travelling costs plus the fixed vehicle utilization-cost. A binary variable X_r is defined for selecting the routes included in the optimal solution. Several customers $i \in I^-$can be visited along a route r for replenishing them with products $k \in K$ picked up at a supply plant $i \in I^+$. A binary parameter a_{irt} is used to indicate whether route $r \in R$ visits ($a_{irt} = 1$) or not ($a_{irt} = 0$) the location $i \in (I^+ \cup I^-)$ during time period $t \in T$.

The minimum number of visits to a given customer $i \in I^-$along the whole planning horizon can be computed by the following expression (Cóccola et al., 2018):

$$\nu_i = \left\lceil \frac{1}{|C|} \sum_{k \in K} \left\lceil \frac{max\left(\sum_{t \in T} dem_{itk} - \left(i^0_{ik} - i^{min}_{ik}\right); 0\right)}{max_c q_c} \right\rceil \right\rceil \; \forall i \in I^- \quad (1)$$

The parameter ν_i is then used in Eq. (2) for imposing a lower bound to the number of routes that will be selected for serving a given customer $i \in I^-$during the planning horizon.

$$\sum_{t \in T} \sum_{r \in R} a_{irt} X_r \geq \nu_i \; \forall \; i \in I^- \quad (2)$$

Each load (unload) operation of product k performed on plant (customer) i forces to update the stock level at site i. The inventory of product $k \in K$ on location $i \in (I^+ \cup I^-)$ at the end of each period $t \in T$ is computed by Eqs. (3) and (4), where Λ_{ikrt} is a continuous positive variable used for determining the quantity of product k picked/delivered from(to) plant(customer) i during period t by feasible route $r \in R$.

$$I_{itk} = \left(i^0_{ik} - \sum_{t' \in T : t' \leq t} dem_{itk} + \sum_{t' \in T : t' \leq t} \sum_{r \in R} \Lambda_{ikrt} \right) \forall i \in I^-, \; t \in T, \; k \in K \quad (3)$$

$$I_{itk} = \left(i^0_{ik} + \sum_{t' \in T : t' \leq t} P_{itk} - \sum_{t' \in T : t' \leq t} \sum_{r \in R} \Lambda_{ikrt} \right) \forall i \in I^+, t \in T, k \in K \quad (4)$$

In addition, Eqs. (5a) and (5b) assure that inventory levels at any site $i \in (I^+ \cup I^-)$ will be larger than the minimum allowed stock i^{min}_{ik} and smaller than the maximum storage capacity i^{max}_{ik}, respectively.

$$I_{itk} \leq i^{max}_{ik} \quad (5.a)$$

$$\forall i \in \left(I^+ \cup I^-\right), t \in T, k \in K$$

$$I_{itk} \geq i^{min}_{ik} \quad (5.b)$$

In order to avoid the supplier from delivering more products that those ones demanded by the customers, Eq. (6) forces that the quantity of inventoried products on all plants (customers) i at the end of the planning horizon must return to the initial existing quantity. The pair of constraints (6) allows balancing the stocks between the facilities of the distribution network, optimizing fulfilment and preventing overselling.

$$\sum_{i \in I^+} i^0_{ik} = \sum_{i \in I^+} I_{itk} \quad (6.a)$$

$$\forall k \in K, t \in T : t = |T|$$

$$\sum_{i \in I^-} i^0_{ik} = \sum_{i \in I^-} I_{itk} \quad (6.b)$$

The allocation of product k to compartment c of vehicle performing route r is defined through Eqs. (7) and (8). Eq. (7) activates the product-to-compartment allocation variable W_{kcr} just in case route $r \in R$ belongs to the optimal solution, i.e. $X_r = 1$. In the other hand, Eq. (8) is a capacity constraint on the quantity of product that can be loaded into each compartment of the truck.

$$\sum_{k \in K} W_{kcr} \leq X_r \; \forall r \in R, c \in R \quad (7)$$

$$\sum_{t \in T} \Lambda_{ikrt} \leq \sum_{c \in C} q_c W_{kcr} \; \forall r \in R, k \in K, i \in I^+ \quad (8)$$

The quantity of product $k \in K$ picked-up (delivered) from (to) plant (customer) i by route r at period t is forced to be zero when either the route r is not selected (X_r=0) or the site i is not visited at period t ($a_{irt} = 0$). This constraint is represented through Eq. (9).

$$\Lambda_{ikrt} \leq \sum_{c \in C} q_c a_{irt} X_r \; \forall i \in \left(I^+ \cup I^-\right), r \in R, t \in T, k \in K \quad (9)$$

Eq. (10) is a balance constraint forcing to discharge all quantities of products $k \in K$ loaded on the vehicle traveling the route $r \in R$.

$$\sum_{t \in T} \sum_{i \in I^+} \Lambda_{ikrt} = \sum_{t \in T} \sum_{i \in I^-} \Lambda_{ikrt} \; \forall r \in R, k \in K \quad (10)$$

For each feasible route $r \in R$, the in-route inventory of product $k \in K$ is computed as follows:

$$I_{rk} = \sum_{t' \in T} \left(\sum_{t \in T} \sum_{i \in I^+} \Lambda_{ikrt} - \sum_{t \in T : t \leq t'} \sum_{i \in I^-} \Lambda_{ikrt} \right) \forall r \in R, k \in K \quad (11)$$

Eq. (11) can be seen as the integral over the whole planning horizon of the actual inventory of product k onboard on route r. In this way, the positive continuous variable I_{rk} is computed as the difference between the total cargos by the period covered by the trip minus the sum of the actual inventory of the product on each covered time-period.

The production activities are represented through Eqs. (12), which impose upper and lower bounds to the produced quantities P_{itk} just in case the production decision given by Y_{itk} is activated. Parameters p^{max}_{ik} *and* p^{min}_{ik} stand for the minimum and maximum production capacity, respectively, of product $k \in K$ in plant $i \in I^+$at time period $t \in T$.

$$P_{itk} \leq Y_{itk} p^{max}_{ik} \quad (12.a)$$

$$\forall i \in I^+, t \in T, k \in K$$

$$P_{itk} \geq Y_{itk} p^{min}_{ik} \quad (12.b)$$

The objective function given by Eq. (13) is to maximize the company profits along the whole planning horizon. The total profit is calculated as the income by sales minus the total cost incurred by the supplier from production, inventory, and distribution activities. The production cost comprises unit production and setup costs while the distribution cost is computed as the route costs plus the in-route inventory costs.

$$MAX\left[\sum_{i\in I^-}\sum_{k\in K}\sum_{r\in R}\sum_{t\in T}p_k\Lambda_{ikrt} - \left(\sum_{i\in I^+}\sum_{k\in K}\sum_{t\in T}\left(c_{ik}^{setup}Y_{itk}+c_{ik}P_{itk}\right) + \sum_{r\in R}\left(c_rX_r+c_k^{in-route}\sum_{k\in K}I_{rk}\right)+\sum_{i\in\left(I^+\cup I^-\right)}\sum_{t\in T}\sum_{k\in K}c_{ik}^{inv}I_{itk}\right)\right] \tag{13}$$

5.1. Valid inequalities

In order to reduce the feasible solution space and expedite the MILP model resolution, the formulation (2)-(13) can be tightened by adding several valid inequalities to be next defined.

- The minimum and maximum number of startups necessary to satisfy the demand of any product $k \in K$ can be computed in advance as follows:

$$y_k^{min} = \left\lceil \frac{\sum_{i\in I^-}\sum_{t\in T} dem_{itk}}{max_{i\in I^-}\left(p_{ik}^{max}\right)} \right\rceil \tag{14.a}$$

$$\forall\, k \in K$$

$$y_k^{max} = \left\lceil \frac{\sum_{i\in I^-}\sum_{t\in T} dem_{itk}}{min_{i\in I^-}\left(p_{ik}^{min}\right)} \right\rceil \tag{14.b}$$

The parameters y_k^{min} and y_k^{max} are then used to define the following valid inequalities:

$$y_k^{min} \le \sum_{i\in I^+}\sum_{t\in T} Y_{itk} \le y_k^{max}\ \forall k \in K \tag{15}$$

- The minimum number of trips departing from plants to customers can be computed as follows:

$$\upsilon^+ = \frac{1}{|C|}\sum_{k\in K}\left\lceil \frac{\sum_{i\in I^-}\sum_{t\in T} dem_{itk}}{max_c q_C} \right\rceil \tag{16}$$

This integer value υ^+ can be used to define the following valid inequality:

$$\sum_{r\in R} X_r \ge \upsilon^+ \tag{17}$$

- In order to avoid alternative optimal solutions, the allocation of any product $k \in K$ to compartment $c \in C$ requires that at least another product k is assigned to compartment $c-1$. This condition is expressed through Eq. (18).

$$\sum_{k\in K} W_{kcr} \ge \sum_{k\in K} W_{kc-1r}\ \forall r \in R \tag{18}$$

6. Generating feasible distribution routes

The resolution of multi-period PIRP problem (2)-(18) becomes cumbersome, in terms of computational burden, when the problem size increased. The practice evidences that CPU processing capacities are quickly depleted when realistic instances of the problem are tried to solve through monolithic approaches. For example, Bard and Nananukul (2010) reported that a small problem involving 3 vehicles, 30 customers, a single product, and a 5-days planning horizon was not solvable to optimality in 90 minutes and optimality gaps between 7% and 10% were generally reported. Also, since feasible routes may be in the order of billions, solving monolithic formulations are no longer considered as a feasible option. This forces to use other solution strategies, such as metaheuristics or decomposition procedures.

By analyzing the structure of formulation (2)-(18), it follows that routing decisions, expressed by the set of feasible routes $r \in R$, can be decoupled from production and inventory decisions. This problem-characteristic allow us to firstly generate a pool of promising routes R' through a CG algorithm as follows.

6.1. Routes generation

CG is a known and effective decomposition technique used for solving large routing problems that was extended to solve also the IRP problem and some of its realistic variations Cóccola et al. (2018). As mentioned above, it is natural to separate inventory and production constraints from routing decisions in the MILP model (2)-(18). After removing production and inventory constraints as well as their associated terms in the objective function, the model is reduced to:

$$MIN\left[\sum_{r\in R} c_r X_r\right] \tag{19}$$

$$\sum_{t\in T}\sum_{r\in R} a_{irt}X_r \ge \upsilon_i\ \forall\, i \in I^- \tag{20}$$

$$\sum_{r\in R} a_{irt}X_r \le 1\ \forall\, i \in I^-, t \in T \tag{21}$$

$$\sum_{r\in R} X_r \ge \upsilon^+ \tag{22}$$

The CG approach solves the routing problem (19)-(22) in an iterative way, considering at each iteration both a master problem restricted to a subset of columns (restricted master problem or RMP) and one or several pricing sub-problems. The procedure starts with a RMP that contains a small number of routes, which can be generated through any heuristic procedure. When binary variables X_r are relaxed, the solution to the problem is a lower bound to the integer routing problem. After finding the optimal solution for the relaxed RMP, the dual variables values σ_i, π_{it} and β from constraints (20) to (22) are passed to the pricing subproblem in order to generate more profitable routes. Afterwards, the new routes computed by the pricing problems are added to the RMP. The iterative procedure continues as far as the optimal solution to the linear master problem cannot be improved with the addition of another column. This condition is true when the pricing problem cannot return a route with a negative reduced cost. The pricing subproblem is defined as follows:

$$MIN\left[C\_ - \sum_{i\in I^-}\sigma_i Y_i - \sum_{i\in I^-}\sum_{t\in T}\pi_{it}Z_{it} - \beta\right] \tag{23}$$

$$\left\{\begin{matrix} S_{ij}+S_{ji} \ge Y_i+Y_j-1 \\ S_{ij}+S_{ji} \le 1 \end{matrix}\right\}\ \forall (i,j) \in\ I^-: i \ne j \tag{24}$$

$$\sum_{i\in I^+} Y_i = 1 \tag{25}$$

$$D_j \ge d_{ij}\left(Y_i + Y_j - 1\right)\ \forall i \in I^+, j \in I^- \tag{26}$$

$$D_j \ge D_i + d_{ij} - M_d\left(1 - S_{ij}\right)\ \forall (i,j) \in I^- : i \ne j \tag{27}$$

$$D\_ \ge D_j + \sum_{i\in I^+} d_{ji} Y_i\ \forall\ j \in I^- \tag{28}$$

$$C\_ = cf + cv\frac{D\_}{speed} \tag{29}$$

$$T_j \ge t^0 + t_{ij}\left(Y_i + Y_j - 1\right)\ \forall i \in I^+, j \in I^- \tag{30}$$

$$T_j \ge T_i + st_i + t_{ij} - M_t\left(1 - S_{ij}\right)\ \forall (i,j) \in I^- : i \ne j \tag{31}$$

$$T\_ \ge T_j + st_j + \sum_{i\in I^+} t_{ij} Y_i\ \forall\ j \in I^- \tag{32}$$

$$T\_ - t^0 \le t^{max} \tag{33}$$

$$\sum_{t\in T} Z_{it} = Y_i\ \forall i \in \left(I^+ \cup I^-\right) \tag{34}$$

$$\left\{\begin{matrix} T_i \ge \sum_{t\in T} (time_{t-1} + a_{it}) Z_{it} \\ T_i \le \sum_{t\in T} (time_t - b_{it}) Z_{it} \end{matrix}\right\}\ \forall i \in I^- \tag{35}$$

$$\left\{\begin{matrix} t^0 \ge \sum_{i\in I^+}\sum_{t\in T} time_{t-1} Z_{it} \\ t^0 \le \sum_{i\in I^+}\sum_{t\in T} time_t Z_{it} \end{matrix}\right\} \tag{36}$$

The objective function (23) defines the reduced cost of the route as the difference between the total traveling cost and the prices collected along the multi-periods route. Binary variable Y_i states that the visiting to location i is included in the route while binary variable Z_{it} specifies the time period t in which the site i is visited. The sites visited along the route are sequenced through the pair of Eq. (24). Binary variable S_{ij} states precedence relationships between any pair of visited customers $(i,j) \in I^-$, being $S_{ij} = 1$ when customer i precedes customer j along the generated route. Eq. (24) force node j to be a predecessor or a successor of node i if both customers are visited ($Y_i = Y_j = 1$). On the other hand, Eq. (25) determines that just one plant $i \in I^+$ can be selected as origin/end of the route. The minimum distance travelled to reach any customer $j \in I^-$ from the selected plant $i \in I^+$ is computed by Eq. (26). In addition, Eq. (27) fixes the accumulated distance travelled by the multi-compartment truck up to each visited customer, using the value the value of sequencing variable S_{ij} and S_{ji}. If $S_{ij} = 1$, customer i precedes customer j. Otherwise, $S_{ji} = 1$ and the reverse statement holds. M_D is the minimum upper bound for variables D_i. The total distance travelled by the truck $D\_$ until returning to the origin plant is determined by Eq. (28) while Eq. (29) computes the total routing cost $C\_$ as the fixed vehicle utilization cost *cf* plus the cost of the in-route time. The timing constraints (30) to (32) are constraints similar to Eq. (26) to (28), but in this case they are defined for determining both the visiting time to every customer i on the route T_i and the end time of the route $T\_$. The time-length of the route $(T\_ - t^0)$ should be lower than t^{max}, as specified Eq. (33). Eq. (34) indicates that the visit to any location $i \in (I^+ \cup I^-)$ must occur just in a single period $t \in T$. If $Z_{it} = 1$, the visiting to customer $i \in I^-$ must occur during period t, as expressed Eq. (35). The parameter $time_t$ specifies the end time of period t while parameters a_{it} and b_{it} determine the time window for visiting customer i during period t. Finally, Eq. (36) fixes the period-activation variable corresponding to the plant $i \in I^+$ from where the vehicle starts/ends the designed tour.

6.2. Valid inequalities for the pricing problem

The routes-generation problem (23)-(36) can be viewed as a multi-period resource-constrained shortest path problem. It is regarded as NP-hard. However, some valid inequalities based on information from time windows may be utilized for expediting the problem resolution. In this way, the following compatibility sets are defined:

Set of successors of customer $i \in I^-$ during period $t \in T$: Node j is said to be a successor of node i if j must not be visited before node i in period t. They are identified by the following set:

$$Suc(i,t) = \left\{\begin{matrix} j \in I^-, t \in T, j \ne i : \left(\left(time_{t-1} + a_{it} + st_i + t_{ij}\right) \le time_t - b_{jt}\right) \wedge \\ \left(\left(time_{t-1} + a_{jt} + st_j + t_{ji}\right) > time_t - b_{it}\right) \end{matrix}\right\}$$

$$\forall\ i \in I^-, t \in T \tag{37}$$

Set of nodes incompatibles with $i \in I^-$ during period $t \in T$: two nodes (*i,j*) that cannot be serviced in the same time t period are called incompatible for such a period. The incompatibility condition for nodes $j \ne i$ is stated by the following set:

$$Inc(i,t) = \left\{\begin{matrix} j \in I^-, j \ne i, t \in T : \left(\left(time_{t-1} + a_{it} + st_i + t_{ij}\right) > time_t - b_{jt}\right) \\ \wedge\left(\left(time_{t-1} + a_{jt} + st_j + t_{ji}\right) > time_t - b_{it}\right) \end{matrix}\right\}$$

$$\forall\ i \in I^-, t \in T \tag{38}$$

These compatibility sets can be used to define the two following valid inequalities:

$$Z_{it} + Z_{jt} + S_{ji} \le 2 \qquad \forall\ (i,j) \in I^-, t \in T, j \in Suc_{it} \tag{39}$$

$$Z_{it} + Z_{jt} \le 1 \qquad \forall\ (i,j) \in I^-, t \in T, j \in Inc_{it} \tag{40}$$

Eq. (39) states that customer $j \in Suc_{it}$ cannot be predecessor of customer i when both are visited during the same period t, i.e. if $Z_{it} = 1$ and $Z_{jt} = 1$ then $S_{ji} = 0$. On the other hand, Eq. (40) represents the incompatibility of visiting two nodes *(i,j)* on the same period t.

6.3. Column Generation algorithm

For realistic instances, feasible routes (columns) may run into billions. Therefore, it is not possible to enumerate all of them. The CG approach handles this complexity by implicitly considering all columns trough the solution of the linear relaxation of the RMP (19)-(22). Every time this linear relaxation is solved, just a subset of all possible routes is considered. Then the optimal values for dual variables associated to constraints (20)-(22) are used in the pricing problem (23)-(40) in order to determine new feasible routes that can reduce the objective function value. Iterations continue until the optimal solution to the linear problem cannot be improved with the addition of another feasible route.

7. Two-stage procedure

As mentioned above, the applicability of the MILP model (2)-(18) to realistic instances of the PIRP problem considering all feasible routes is either impossible or yield poor solutions. Every time a new route is added to the PIRP formulation, new associated constraints must be added as well. Note that the higher the number of routes, the largest solution space and more complex and intractable the MILP model resolution. To overcome this drawback, this paper proposes a solution approach based on a strategy of first computing a set of feasible distribution routes *R'* through the column generation algorithm and then using this pool of columns for solving the MILP model (2)-(18). The general structure of this two-stage procedure is depicted in Fig. 2.

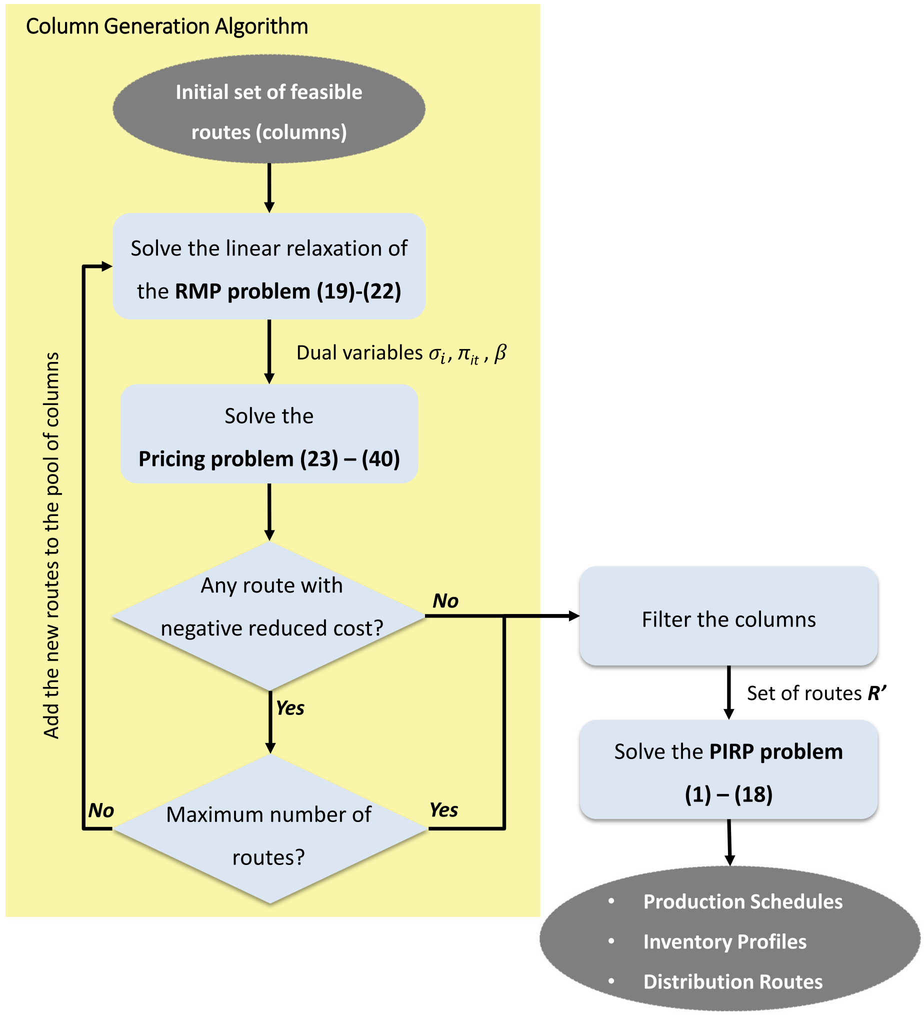

Fig. 2. The two-stage procedure for solving the PIRP problem.

The procedure receives as input an initial pool of columns. So, for each time period $t \in T$, routes $i - j - i$ starting from any plant $i \in I^+$ and going to any customer $j \in I^-$ are heuristically generated. Then, the CG algorithm starts to iterate until not a new route with a negative reduced cost can be found or until the maximum number of generated routes is achieved. Since the number of generated routes by the CG approach can easily run in thousands, the procedure is configured to maintain the number of useful routes bounded through two simple actions: (i) *early termination*; given the tailing-off effect (Lübbecke and Desrosiers, 2005) consisting on a slow convergence of to the optimal solution to the RMP, the generation procedure ends after a stated number of master-slave iterations or after generating a given number of columns and (ii) *routes-filtering*; to further reduce the pool of columns feed to the PIRP problem, columns are ranked in decreasing order according to the following index:

$$vi_r = \frac{\sum_{i \in I^-} \sum_{t \in T} a_{irt}}{c_r} \; \forall r \in R' \tag{41}$$

The index-value $vi_r$is related to the number of visits per unit of routing cost. A large number would be indicative of an attractive route and a low value indicates a relatively expensive per-visited-customer route. Therefore, routes with a vi_r value below a given threshold are eliminated from the columns pool R' used to solve the PIRP problem. The problem, conversely, may be fed with a maximum number of columns ranked in decreasing value of vi_r.

After the routes generation phase and the optional filtering stage, the pool of generated routes R' replaces the set of feasible routes R in the formulation (2)-(18). As result the procedure returns: (i) the production schedule, (ii) the inventory profiles, and (iii) the selected distribution routes.

8. Computational results

The procedure performance was first tested by solving a set of PRP benchmark instances proposed by Archetti et al. (2011). Then several instances generated from data originally proposed by Marchetti et al. (2014) and later modified and tackled by Cóccola et al. (2018) were solved though the proposed solution strategy. The algorithm and underlying models were codified in GAMS 24.8.5 using CPLEX 12.6.3 as the MILP solver. The hardware was an Intel® Core ™ i7, 16 GB 2.80 GHz desktop PC.

8.1. Archetti et al. benchmark instances

Archetti et al. (2011) compared maximum level and order-up-to-level policies in the context of the PRP considering the

Table 1
Solutions to the first class of Archetti's instances with 19 retailers.

Instance	Archetti's B&C algorithm	Archetti's MIP-heuristic algorithm	Two-stage procedure	% improvement	Total CPU time (seconds)
1	50168	50196	52465	−4,58	601,7
2	50509	51541	51163	−1,29	112,3
3	54321	55127	52466	3,41	583,6
4	104807	105756	105494	−0,66	83,6
5	105966	106598	106511	−0,51	118,4
6	112428	113307	107209	4,64	419,2
7	41151	41590	41170	−0,05	124,7
8	41566	42377	42359	−1,91	359,3
9	–	44404	43237	2,63	624,3
10	93313	96379	93309	0,00	114,5
11	95778	98202	93396	2,49	151,4
12	–	101926	93875	7,90	124,6
13	54751	55808	55052	−0,55	189,5
14	55929	56967	56821	−1,59	1558
15	59837	60800	59835	0,00	1538.1
16	109423	110738	109713	−0,27	109,5
17	111406	112878	111782	−0,34	352.3
18	118049	118542	114081	3,36	885,5
19	44402	45304	44949	−1,23	284,6
20	45667	46594	46620	−2,09	459,9
21	49459	49682	48953	1,02	601,7
22	96937	100143	98064	−1,16	140,3
23	99812	101244	98623	1,19	164,9
24	–	106932	100153	6,34	317,7

single-product, single-period-routes variant of the problem. In-route inventory costs were not considered in those benchmark problems. The authors presented benchmark instances available at the authors' website (http://or-brescia.unibs.it/instances). Problem characteristics are detailed in the cited paper. They are regarded as similar, but simpler, than the problem features considered in this work and consequently, it was decided to use some of these standard instances as a test-bed to evaluate the performance of our algorithm. Archetti et al. considered a particular PRP with constant demands and an un-capacitated production plant. Moreover, the distribution problem disregards the features of multi-period routed and multi-compartment vehicles. Time windows for delivering are also not considered. These differences imply that several modifications must be introduced to our algorithm in order to deal with such benchmark instances. Consequently, the RMP, the pricing problem and the PIRP were simplified and rewritten (see supplementary information) in order to match the same assumptions. Due to the constraint imposed by Archetti et al., which specifies that each retailer cannot be visited by more than one vehicle at each time instant, the binary variable X_r used in the original formulation of PIRP problem becomes X'_{tr} (see supplementary information). Archetti et al. considered four classes of 24 instances with 19, 50 and 100 retailers and an unlimited fleet. We solved the first class of instances with 19 and 50 retailers because routing problems with 100 visited sites and no time windows constraints are beyond CG-approaches capabilities to optimally solve the problem (Archetti et al. heuristically generated these no-time-windows-constrained single period routes). Tables 1 and 2 compare the best solutions found by our algorithm for the first class of instances with 19 and 50 retailers, respectively, against the best solutions reported by Archetti et al. As a result, an improvement average of 0.7% over the best solution found for the 19 retailers instances was achieved by the proposed two-stage algorithm while for instances with 50 retailers, the objective value is 6% worsened on average. This is because of the poor performance exhibited by the CG algorithm for generating routes in a shortest path problem without resource constraints. In addition, the PIRP phase cannot close the integrality GAP within the CPU time limit of 3600 seconds, as it is observed in Table 2.

8.2. PIRP instances

The solution procedure was also tested by solving several realistic instances generated from data originally proposed by Marchetti et al. (2014) and later tackled by Cóccola et al. (2018). This case study involves the production and distribution, via two-compartment trucks, of two products from up to three plants in order to supply up to fifty customers over two time-horizons of seven and fourteen days, respectively. Customers are randomly placed in different geographical locations and all facilities in the network are defined by the *(X, Y)* coordinates in the Euclidean plane (See Fig. 3). Distances (in miles) between plants and customers are computed from *(X, Y)* coordinates as Euclidean distances.

Data taken from instances proposed by Marchetti et al. (2014) can be downloaded from http://dx.doi.org/10.1016/j.compchemeng.2014.06.010 while data taken from Cóccola et al. (2018) can be downloaded from https://doi.org/10.1016/j.compchemeng.2018.08.004. For any facility, plant or customer, the parameter *"redline"* defined by Marchetti et al. (2014) represents the safety stock at the end of any time-period while *"maximum"* stands for the maximum allowed inventory level. The plants host an unspecified number of identical two-compartment vehicles with capacity $q_c = 630$ ft^3 per compartment. Unlike Cóccola et al. (2018), which assumed that daily production parameters were known in advance and that just daily-routes could be used, this work releases the production decisions, which will be determined by the solution-procedure. More useful and harder multi-period routes are also utilized.

Several sets of instances are generated and solved in this work. Nominal *"N-instances"* instances are defined by introducing the parameters values reported in Table 3. The impact of time windows (TW) on solutions is studied by the introduction of TW constraints in order to define a set of time-windows constrained *"TW-instances"*. The time windows data for this set of instances are specified as supplementary information. Note that in multi-period routes, TW must apply to each period of the planning horizon. Later, setup and production costs are incremented and the minimum and maximum productions levels of each plant are also increased with respect to the production levels presented in

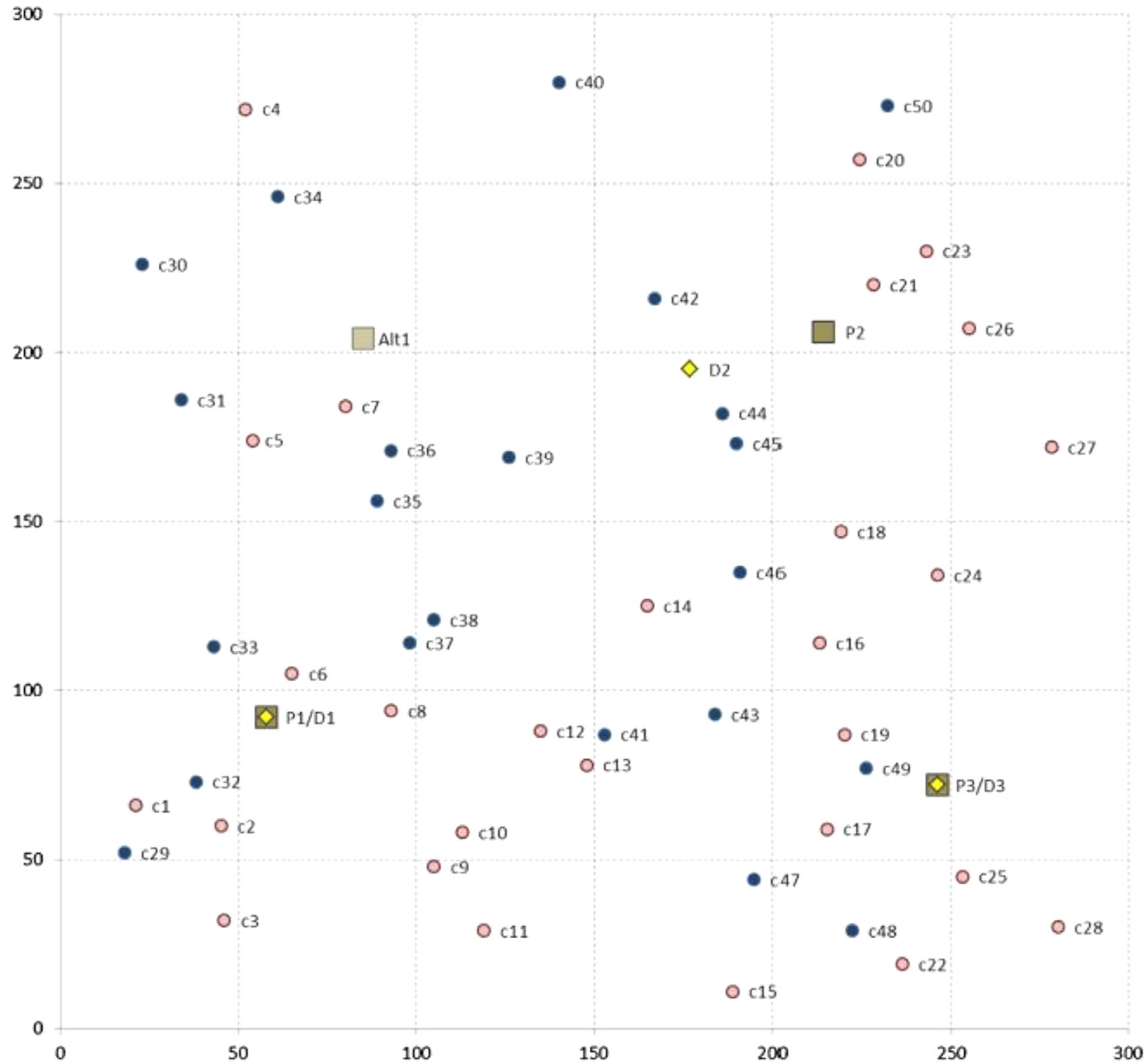

Fig. 3. Geographical distribution of plants and customers (reprinted from Marchetti et al., 2014).

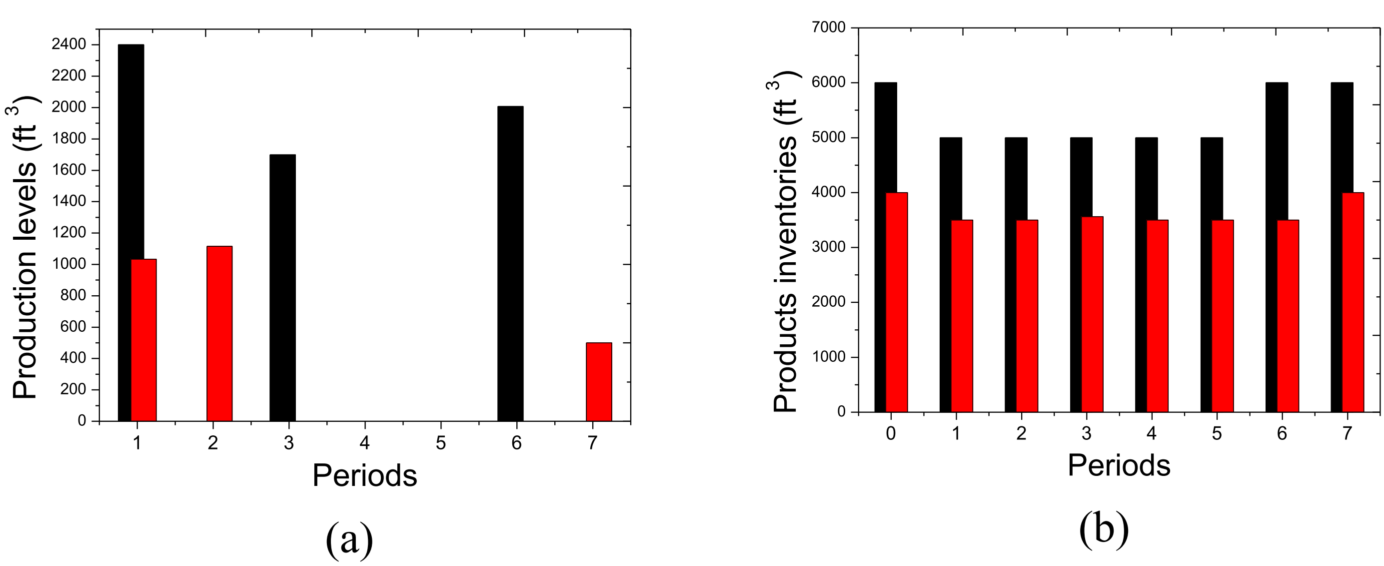

Fig. 4. Production schedule and inventory-profiles for both products on the solution to the instance 1-7-5-7-N.

Table 2
Solutions to the first class of Archetti's instances with 50 retailers.

Instance	Archetti's MIP-heuristic algorithm	Two-stage procedure	% improvement	PIRP integrality gap (%)
1	115646	122824	−6,21	10.0
2	117470	123193	−4,87	7,5
3	118379	127567	−7,76	6.9
4	237301	244707	−3,12	4,8
5	237537	245217	−3,23	9,7
6	240465	252548	−5,02	5,6
7	94967	100642	−5,98	8,3
8	95657	100654	−5,22	7,6
9	95455	102034	−6,89	8,1
10	214312	225941	−5,43	8,4
11	215009	224464	−4,40	9.0
12	215700	227840	−5,63	9.1
13	127361	141443	−11,06	8.9
14	128114	138614	−8,20	8.3
15	130398	144255	−10,63	11.5
16	250602	268367	−7,09	7.7
17	250010	267843	−7,13	8.9
18	252166	273177	−8,33	8.8
19	101829	112959	−10,93	11,3
20	104811	111249	−6,14	12.0
21	107367	116939	−8,92	9.3
22	222744	237272	−6,52	11.0
23	227246	239672	−5,47	12.6
24	226050	239612	−6,00	7.3

Table 3
Vehicles, inventory and production parameters for realistic instances.

Vehicle parameters		Inventory Parameters		Products and production parameters					
Capacities (ft^3)		Inventory costs ($/day ft^3)		Setup costs ($/day)					
C_1: 630	C_2: 630	In plants			*Plant*	$Plant_2$	$Plant_3$		
		k_1: 0.020	k_2: 0.020	k_1	800	700	500		
	In Customers								
		k_1: 0.035	k_2: 0.035	k_2	800	800	500		
Routing parameters	In route inventory costs ($/*day* ft^3)	Production costs ($/$ft^3$)							
cf: $ 150 *cv*: $52.1/h	k_1: 0.1	k_2: 0.1		$Plant_1$	$Plant_2$	$Plant_3$			
st^+: 0.2 day; st^-: 0.1 day		k_1	3.0	2.5	3.0				
t^{max}: 2.0 days		k_2	3.0	3.0	3.5				
		Inventory levels (ft^3)				*Production levels (ft^3)*			
Speed: 300 miles/day		$Plant_1$	$Plant_2$	$Plant_3$		$Plant_1$	$Plant_2$	$Plant_3$	
Product prices									
K_1: $ 6/ft^3	K_2: $ 8/ft^3								
		I_{k1}^{min}	5000	3000	4000	p_{k1}^{min}	200	200	200
		I_{k1}^{max}	18000	12000	14000	p_{k1}^{max}	2400	1000	2400
		I_{k2}^{min}	3500	4000	3000	p_{k2}^{min}	100	100	100
		I_{k2}^{max}	12000	9000	10000	p_{k2}^{max}	1200	500	1200

Table 4
Initial inventory levels on plants for type-1 and type-2 instances.

	Type-1 instances (ft^3)			Type-2 instances (ft^3)		
	$Plant_1$	$Plant_2$	$Plant_3$	$Plant_1$	$Plant_2$	$Plant_3$
I_{k1}^0	6000	3500	4500	10000	8500	6500
I_{k2}^0	4000	4500	4000	8000	6500	7000

Table 3 in order to define high-scale-plants *"HS-instances"*. Finally, the initial inventory level at each site is reduced to the minimum inventory level (i.e. $i_k^0 = i_k^{min}$) in order to define the so called minimum-inventory *"MI-instances"*. All defined instances are further classified into types "1" and "2", like e.g. N-1 and N-2. Type-1 problems have a short planning horizon (7 days) while type-2 problems feature a longer planning horizon (14 days).

Initial inventory levels for all set of instances (except MI-instances) on both type-1 and type-2 planning horizons are presented in Table 4.

The solved instances are labelled according to the quantity of plants, the number of customers consuming the first product k_1, the number of customers consuming the second product k_2 and the length of planning horizon *th*. For example, the instance $|I^+| = 1$, $|I^-|_{k1} = 7$, $|I^-|_{k2} = 5$ and $th = 7$ feature an instance involving $Plant_1$, the first seven customers consuming the product k_1 (i.e. $i_1, \dots i_7$), the first five customers consuming k_2 (i.e. $i_{29}, \dots i_{33}$) and a time-horizon of seven days.

8.3. Assessing the effect of valid inequalities

The effect of valid inequalities in the resolution performance is briefly tested by solving four small scale instances with and without valid inequalities (14) to (18). The computational results obtained are summarized in Table 5.

From Table 5, it follows that the CPU time saving was roughly 15% for the first three solved instances. The fourth instance was not solved to optimality within the 3600 s CPU time-limit without

Table 5
Effect of valid inequalities on the resolution of small scale instances.

Instance				Without valid inequalities		With valid inequalities		
$\|I^+\|$	$\|I^-\|_{k1}$	$\|I^-\|_{k2}$	*Th*	*Objective function*	*CPU time*	*Objective function*	*CPU time*	*Ratio*
1	7	5	7	15408.0	222.1	15408.0	197.1	0.887
2	7	5	7	14448.7	40.7	14448.7	35.4	0.869
1	7	5	14	45076.4	361.7	45076.4	302.3	0.836
2	7	5	14	20088.7	3600*(2.1%)	20088.7	3520.9	–

* CPU time limit reached (Integrality gap).

Table 6
Results from the evaluation of the columns filtering technique.

Instance				Thresholds		Objective function	CPU time (s)			
$\|I^+\|$	$\|I^-\|_{k1}$	$\|I^-\|_{k2}$	*th*	*Columns generated*	*Columns accepted*		*RMP*	*Pricing Problem*	*PIRP*	*Total*
1	7	5	7	No limit (186)	No limit (186)	15408.0	0.1	15.5	165.7	181.4
				No limit (186)	150	15392.7	0.1	15.5	92.6	108.2
				No limit (186)	100	15274.0	0.1	15.5	31.6	47.2
				150	150	15388.6	0.1	7.1	29.6	36.7
				100	100	15315.4	0.1	4.6	5.8	10.5
2	7	5	7	No limit (186)	No limit (284)	14448.7	0.1	22.4	76.5	99.1
				No limit (186)	200	14448.7	0.1	21.6	54.4	76.1
				No limit (186)	150	14311.3	0.1	21.6	300.9	322.6
				No limit (284)	100	14153.9	0.1	21.2	61.9	83.2
				150	150	14144.9	0.1	1.1	0.9	2.1
1	7	5	14	327	327	45076.4	0.2	169.4	302.3	471.9
				327	300	Infeasible	0.2	168.8	–	–
				325	325	45076.4	0.1	132.1	314.2	446.5
2	7	5	14	No limit (463)	No limit 463)	20088.7	0.1	91.1	3520.9	3612.1
				No limit 463)	400	20088.7	0.1	90.7	3004.2	3095.0
				No limit (463)	300	Infeasible	0.1	90.8	–	–
				350	350	20088.7	0.1	81.7	3054.0	3135.8

valid inequalities but these additional constraints allowed finding the optimal solution within such a limit. These results demonstrate that the use of valid inequalities is helpful in reducing the CPU solving-time but their effect is quite moderate.

8.4. Assessing the effect of the columns filter

In order to validate the heuristic filtering technique, which aims at downsizing the solution search space of the PIRP problem, some instances were solved several times considering a different number of columns to be generated and accepted with the aim of comparing objective function values and CPU times. The threshold of accepted columns was progressively downsized and the results were evaluated. Results are summarized in Table 6. This table shows, for each solved instance, the number of columns generated by the CG algorithm and how many of these routes were accepted and incorporated into the set of feasible routes of the PIRP problem. In addition to the objective function value, the accumulated computational times consumed by each model resolution are also given in this table.

From Table 6, it follows that the objective function value seems not to be affected by a change in the number of columns considered in the PIRP and just degrades when the filtering threshold is near the infeasibility level given by a too low number of columns available in the pool. On the other hand, the CPU time significantly downs when the number of columns is reduced.

8.5. Results and discussion

After validating the usefulness of valid inequalities and the heuristic filtering technique, we solved all instances described in subSection 8.2 with numerical settings detailed in Table 7.

Data about billing, costs and profits of the best solution as well as the number of selected routes for each type-1 instance are reported in Table 8.a while data about computational costs incurred to provide solutions to these instances are presented in Table 8.b. The same information about type-2 instances are respectively reported in Tables 9.a and 9.b. In Tables 8.b and 9.b, the column *#Cols* shows the number of columns fed to the PIRP while the integrality gap reported by the resolution of the problem model is given in the column *Gap* whenever the maximum CPU time for the PIRP is reached and the integrality gap is not closed. From results of Tables 8.a and 9.a, it can be observed that the number of used vehicles, for a given instance-size, remain quite similar in spite of changes of instances-parameters between instances types. Note also that profits don't change significantly as a function of changes on these parameters.

Table 7
Setting options for the solution algorithm.

Options	
MIP solver	CPLEX 12.6.3
Maximum CPU time per master-slave iteration (s)	120
Maximum CPU time for the PIRP problem (s)	3600
Multiple columns generated per iteration	Yes
Filtering of columns visiting the same subset of customers	Yes
Time-windows reduction and pre-processing	Yes
Maximum number of generated columns (generation stage)	2500
Maximum number of accepted columns (PIRP problem)	1000

The conclusions that can be extracted from the information presented in Tables (8) and (9) are the following: (i) instances with a given number of customers and plants involving a 14 days planning horizon are much harder to solve than the instances involving a 7 days planning horizon. Note that the integrality gap of the PIRP was closed in all type-1 instances within the 3600 *s* maximum CPU time while in almost a half of type-2 instances this gap was not closed; (ii) in some cases, the CPU times incurred for solving large instances of the PIRP are smaller than solution

Table 8.a
Best solutions found for Type-1 instances (*th*= 7).

Ins	*Plants*	*Customers*		*Billing($)*	*Costs ($)*					*Profit($)*	*#Routes*
	$\lvert I^{+}\rvert$	$\lvert I^{-}\rvert_{k1}$	$\lvert I^{-}\rvert_{k2}$		*Routing*	*Inventory*	*In-route inventory*	*Production setup*	*Production*		
N-1	1	7	5	57834.0	5870.9	4424.9	1062.2	4800.0	26268.0	15408.0	9
N-2	1	14	11	123990.0	12829.3	7249.4	4139.1	8800.0	55248.0	35724.2	18
N-3	2	7	5	57834.0	6479.2	5702.6	535.5	5400.0	25238.0	14448.7	10
N-4	2	14	11	123990.0	13196.0	8607.2	3213.2	10100.0	53748.0	35125.5	17
N-5	2	21	16	189144.0	21833.5	11120.1	6347.3	16900.0	81286.0	51657.1	30
N-6	3	28	22	266892.0	29175.2	16324.9	7410.4	19600.0	118883.0	75498.5	43
TW-1	1	7	5	57834.0	5783.4	4453.9	1255.9	4800.0	26268.0	15272.8	8
TW-2	1	14	11	123990.0	14591.8	8175.9	2780.2	8800.0	55248.0	34394.7	17
TW-3	2	7	5	57834.0	5783.4	5521.6	1277.6	4800.0	26268.0	14183.4	8
TW-4	2	14	11	123990.0	14179.3	8419.0	3710.5	10100.0	53748.0	33842.2	18
TW-5	2	21	16	189144.0	23250.3	11269.2	6685.1	12900.0	81286.0	53753.4	29
TW-6	3	28	22	266892.0	30208.8	16364.0	7432.5	18200.0	119383.0	75303.7	42
HS-1	1	7	5	57834.0	6137.5	4698.1	600.4	6400.0	26268.0	13730.0	10
HS-2	1	14	11	123990.0	12979.3	7369.8	3709.9	14400.0	55248.0	30283.1	17
HS-3	2	7	5	57834.0	5866.7	5699.9	1013.0	6400.0	26268.0	12589.4	9
HS-4	2	14	11	123990.0	13504.3	8608.7	3246.7	15400.0	55274.5	30775.8	16
HS-5	2	21	16	189144.0	22183.6	11394.1	5495.8	23000.0	80051.5	47019.0	27
HS-6	2	28	22	266892.0	34333.8	14633.2	9016.2	33800.0	112220.0	62888.8	38
HS-7	3	28	22	266892.0	29758.7	16516.1	6499.3	27400.0	116333.0	70385.0	43
MI-1	1	7	5	57834.0	6137.5	4491.4	729.2	4800.0	26268.0	15407.8	10
MI-2	1	14	11	123990.0	13004.3	7039.2	1176.0	8800.0	55248.0	35370.9	18
MI-3	2	7	5	57834.0	5854.2	5338.7	4527.6	4800.0	26268.0	14397.1	9
MI-4	2	14	11	123990.0	13270.9	8281.7	3746.0	10100.0	53988.0	34603.4	19
MI-5	2	21	16	189144.0	22279.4	10452.6	8068.8	16900.0	81451.5	49991.7	28
MI-6	3	28	22	266892.0	30258.7	15660.3	8399.3	18900.0	119326.0	74339.7	41

Table 8.b
Computational costs for solutions reported in Table 8.a.

Ins	*Plants*	*Customers*		*CPU TIME (seconds)*					
	$\lvert I^{+}\rvert$	$\lvert I^{-}\rvert_{k1}$	$\lvert I^{-}\rvert_{k2}$	*#Cols*	*RMP*	*PP*	*PIRP*	*Total*	*Gap*
N-1	1	7	5	186	0.1	15.5	165.7	181.4	–
N-2	1	14	11	495	0.3	2648.4	301.3	2950.0	–
N-3	2	7	5	265	0.1	16.8	34.9	51.8	–
N-4	2	14	11	734	0.3	2478.1	321.6	2799.7	–
N-5	2	21	16	808	0.3	2554.6	305.5	2860.4	–
N-6	3	28	22	1050	0.6	3755.4	108.0	3864.0	–
TW-1	1	7	5	127	0.1	7.9	1.4	9.3	–
TW-2	1	14	11	298	0.1	299.8	27.9	327.8	–
TW-3	2	7	5	207	0.1	8.1	2.2	10.4	–
TW-4	2	14	11	489	0.2	392.9	109.7	502.7	–
TW-5	2	21	16	664	0.3	2253.0	227.4	2480.7	–
TW-6	3	28	22	1169	0.5	3755.3	59.2	3815.0	–
HS-1	1	7	5	175	0.1	8.5	235.8	244.3	–
HS-2	1	14	11	493	0.2	2254.2	303.1	2557.5	–
HS-3	2	7	5	265	0.1	17.5	300.8	318.4	–
HS-4	2	14	11	713	0.3	2627.8	401.6	3029.6	–
HS-5	2	21	16	835	0.4	3004.4	303.5	3308.3	–
HS-6	2	28	22	780	0.5	4504.0	82.3	4586.8	–
HS-7	3	28	22	1108	0.5	3753.2	317.5	4071.2	–
MI-1	1	7	5	200	0.1	22.1	38.9	61.2	–
MI-2	1	14	11	413	0.2	1802.4	252.9	274.0	–
MI-3	2	7	5	268	0.2	20.9	301.8	2104.4	–
MI-4	2	14	11	632	0.3	2282.3	322.2	2604.9	–
MI-5	2	21	16	722	0.6	3753.9	504.5	4259.0	–
MI-6	3	28	22	1189	0.6	6005.0	308.0	6131.6	–

times used to solve some smaller ones. E.g.; PIRP problems for all ($|I^{+}| = 3$, $|I^{-}|_{k1} = 28$, $|I^{-}|_{k2} = 22$, $th = 14$) scenarios were solved within the 3600 s time limit but several smaller PIRP problems reached such a limit. In such cases, there is less freedom to allocate production activation decisions. I.e., production takes place in most days and therefore, there are less possible combinations than in instances where a few production-activation decisions take place; (iii) the biggest integrality gap is just over 5% in a hardest midsize type-2 instances. That's remarkable, given the existences of multiple optimal solutions implicit in the nature of the PIRP. The search for additional constraints aimed at improving the resolution efficiency, although out of the scope of this paper, deserves further research.

8.6. Illustrating an example

In order to illustrate the information provided by the solution algorithm, the solution to the instance ($\lvert I^{+}\rvert = 1$, $\lvert I^{-}\rvert_{k1} = 7$, $\lvert I^{-}\rvert_{k2} = 5$, $th = 7$) is depicted in Figs. 4 and 5. Fig. 4.a shows the production levels while Fig. 4.b details the inventory time-evolution for both products in the storing devices of the plant. In addition, Fig. 5 shows the evolution of inventories in each serviced customer. Note that a mix of order-up-to-level and maximum-level replenishment operations was recorded.

Due to space constraints, solution data about production schedules, inventories profiles (both on the plant and the customers), and delivery routes are detailed as supplementary information.

Table 9.a
Best solutions found for Type-2 instances (*th*= *14*).

Ins	Plants	Customers		Billing($)	Costs ($)					Profit($)	#Routes
	$\vert I^+\vert$	$\vert I^-\vert_{k1}$	$\vert I^-\vert_{k2}$		Routing	Inventory	In-route inventory	Production setup	Production		
N-1	1	7	5	115296.0	9297.5	9690.9	7509.1	8800.0	34922.0	45076.4	20
N-2	1	14	11	246432.0	20127.6	16611.7	16110.7	17600.0	109731.0	65651.0	39
N-3	2	7	5	115296.0	13750.0	13336.8	7937.5	9300.0	50883.0	20088.7	19
N-4	2	14	11	246432.0	30025.1	16441.6	16555.3	17600.0	109731.0	56079.0	38
N-5	2	21	16	378948.0	46141.8	26316.1	25638.5	33800.0	162878.0	84173.6	61
N-6	3	28	22	529848.0	55966.7	36954.5	35314.8	38400.0	234494.5	12877.5	86
TW-1	1	7	5	115296.0	8867.5	9665.4	7761.4	8800.0	52383.0	27818.9	19
TW-2	1	14	11	246432.0	20415.1	16247.8	7853.6	16800.0	71948.0	9960.9	39
TW-3	2	7	5	115296.0	12829.2	9284.5	16587.5	9900.0	49883.0	25005.6	19
TW-4	2	14	11	246432.0	29008.4	19866.1	17225.2	20100.0	106231.0	54001.3	40
TW-5	2	21	16	378948.0	45616.8	26276.0	26004.8	33800.0	162878.8	84372.4	60
TW-6	3	28	22	529848.0	54758.3	37077.7	36020.8	38400.0	234494.5	129096.7	85
HS-1	1	7	5	115296.0	9320.0	9709.1	7452.3	12800.0	34922.0	41092.5	20
HS-2	1	14	11	246432.0	20067.7	16225.4	16951.7	27200.0	73154.0	92833.2	37
HS-3	2	7	5	115296.0	14145.9	13535.0	7477.1	13800.0	49383.0	16955.1	20
HS-4	2	14	11	246432.0	29575.2	19085.0	16689.9	27800.0	103801.0	47620.9	38
HS-5	2	21	16	378948.0	46991.8	26239.7	25031.1	44200.0	158883.5	77601.9	62
HS-6	2	28	22	529948.0	67600.0	33301.0	36823.8	33000.0	222865.0	136258.2	84
HS-7	3	28	22	529948.0	56850.4	36588.6	36500.4	55400.0	227594.5	117015.1	83
MI-1	1	7	5	115296.0	6617.5	9198.3	6979.8	8800.0	52383.0	31317.4	22
MI-2	1	14	11	246432.0	14557.6	15302.6	17944.9	17600.0	109731.0	71295.9	43
MI-3	2	7	5	115296.0	12558.4	10758.9	8398.9	8700.0	52051.5	22828.3	18
MI-4	2	14	11	246432.0	31216.7	19307.6	16838.6	29000.0	102743.5	47325.6	40
MI-5	2	21	16	378948.0	46466.8	26244.3	25313.3	44200.0	158883.5	77480.1	61
MI-6	3	28	22	529848.0	57933.3	33698.5	38282.9	37500.0	236467.5	125965.4	86

Table 9.b
Computational costs for solutions reported in Table 9.a.

Ins	Plants	Customers		CPU TIME (seconds)					
	$\vert I^+\vert$	$\vert I^-\vert_{k1}$	$\vert I^-\vert_{k2}$	#Cols	RMP	PP	PIRP	Total	Gap
N-1	1	7	5	306	0.1	110.4	302.4	412.8	–
N-2	1	14	11	807	1.3	3607.8	3600*	7209.1	2.1
N-3	2	7	5	465	0.1	161.7	307.2	469.0	–
N-4	2	14	11	672	0.9	2780.9	3600*	6381.8	2.2
N-5	2	21	16	1063	1.0	502.0	3600*	4103.0	2.6
N-6	3	28	22	2160	0.6	6005.0	224.4	6250.0	–
TW-1	1	7	5	229	0.1	13.0	159.9	173.0	–
TW-2	1	14	11	425	0.1	416.1	3600*	4016.2	1.0
TW-3	2	7	5	380	0.1	10.9	978.1	989.1	–
TW-4	2	14	11	780	0.1	353.3	3600*	3953.4	3.2
TW-5	2	21	16	1344	0.6	4506.9	3600*	8107.4	3.4
TW-6	3	28	22	2137	0.7	6754.4	3600*	7064.8	–
HS-1	1	7	5	325	0.1	193.2	1558.7	1752.0	–
HS-2	1	14	11	759	1.2	4147.7	3600*	5856.2	1.5
HS-3	2	7	5	463	0.1	91.8	3600*	3692.0	5.0
HS-4	2	14	11	1007	0.6	2255.6	3600*	7749.0	2.1
HS-5	2	21	16	1036	0.5	4505.6	3600*	8106.1	4.8
HS-6	2	28	22	1487	0.6	5251.1	205.5	5460.1	–
HS-7	3	28	22	2257	0.6	5257.9	150.1	5408.6	–
MI-1	1	7	5	252	0.1	130.5	302.9	433.5	–
MI-2	1	14	11	757	0.5	4234.6	370.8	4605.9	–
MI-3	2	7	5	482	0.2	137.6	303.3	441.1	–
MI-4	2	14	11	1123	0.5	5248.5	3600*	8849.0	5.1
MI-5	2	21	16	1394	0.6	5256.4	3600*	8857.0	4.5
MI-6	3	28	22	2137	0.7	6753.9	447.6	7202.2	–

* CPU time limit reached.

9. Conclusion

This paper researched the optimal planning of production, inventory and distribution of products requiring non-negligible preservation efforts and costs. It is our third work on a research line aimed at the integration of production, inventory and delivery of chemical fluids. The paper presented a decomposition algorithm for planning, over a multi-period time-horizon, the production, inventory and distribution of several industrial products. Forecasted demands are assumed as data known in advance while quantities to be produced at each plant and the consequently inventory profiles at all facilities in the distribution network are decisions which must be optimized by the solution procedure. The proposed algorithm determines, in a first stage, the best multi-period routes for distributing several generic products from plants to customers through a CG mechanism; next, this set of routes is used for solving the PIRP problem. The goal was to decouple routing decisions from production and delivering decisions in order to obtain near-optimal solutions in practical computational times. It is worth to remark that multi-period routes considered in this work constitute a scarcely researched issue in the literature referred to the joint planning of production, inventorying and distribution. From a

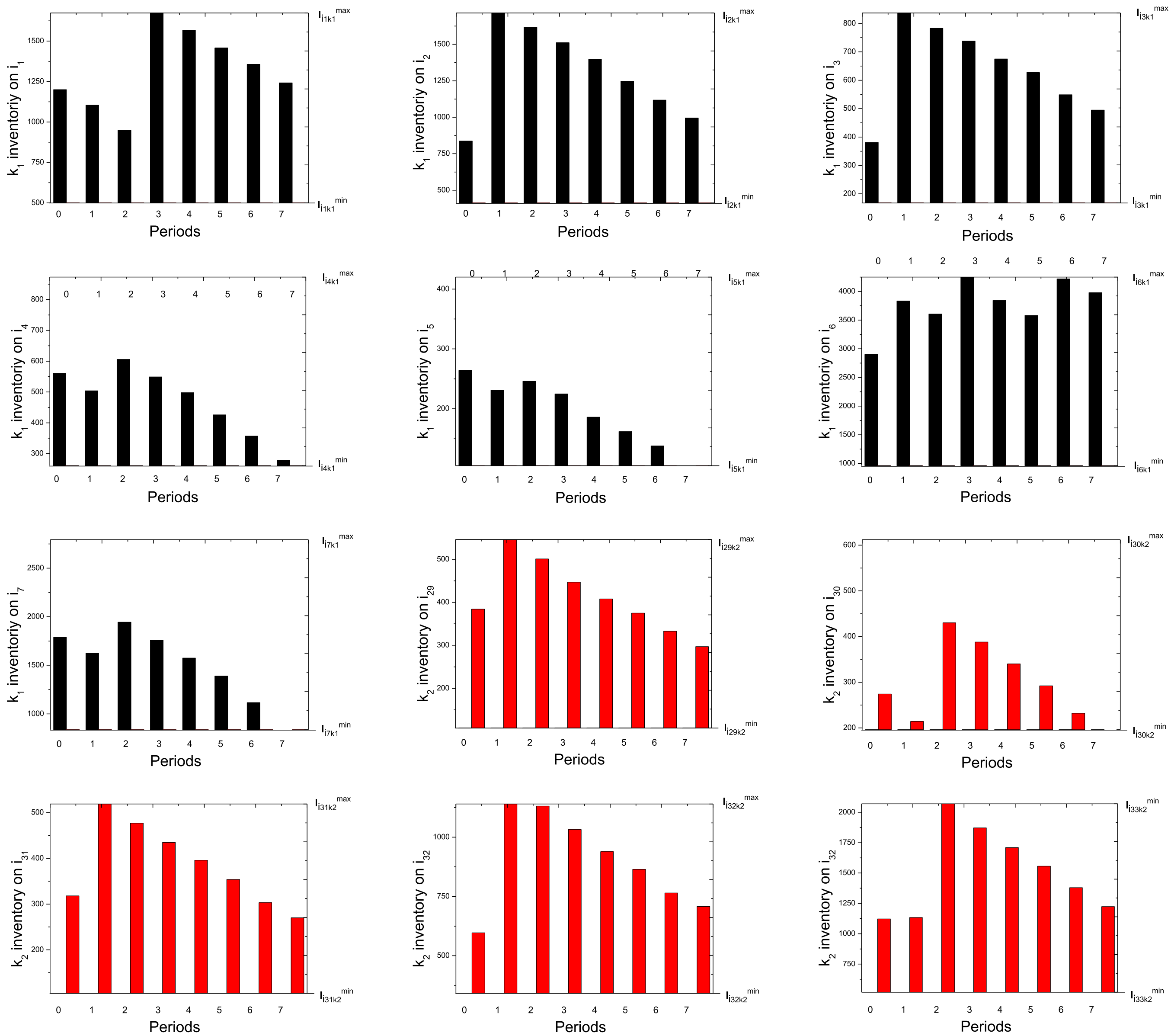

Fig. 5. Inventory profiles on customers for the solution to the instance 1-7-5-7-N.

routing point of view, the solution provided by the algorithm indicates the time to serve a given customer, the quantity of products to deliver to the visited customers and the optimal sequence of visited customers by each vehicle-route. The procedure is able also to consider the allocation of several products on the same vehicle by fixing the quantity and type of product transported on each vehicle-compartment. From a production point of view, the algorithm computes the quantity of products made during each period of the planning horizon, i.e. the production schedule. This is of utmost importance because storing costs may be an important fraction of total costs when dealing with perishable products that must be preserved. It is worth noting that in-route inventories here considered are neglected in the scant bibliography referred to the joint planning of production, inventory and distribution and also in the bibliography referred to the IRP and its variants. This term, although negligible in the distribution of non-perishable products may be important when considering the distribution of perishables because of the need of preserving the in-route cargo. The proposed solution procedure has been used to solve a set of benchmark instances taken from the literature and several realistic instances of four different scenarios featuring two different planning-horizons and several numbers of customers and plants. In all examples, the solutions were obtained with relatively moderate computational effort.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.compchemeng.2019.106690.

CRediT authorship contribution statement

Mariana E. Cóccola: Conceptualization, Methodology, Software, Writing - review & editing. **Carlos A. Méndez:** Conceptualization, Methodology, Software, Writing - review & editing. **Rodolfo G. Dondo:** Conceptualization, Methodology, Software, Writing - review & editing.

References

Adulyasak, Y., Cordeau, JF., Jans, R., 2014. Formulations and branch and cut algorithms for multi-vehicle production and inventory routing problems. INF J. Comput. 26 (1), 103–120.

Adulyasak, Y., Cordeau, J.-F., Jans, R., 2015. Production routing problem: a review of formulations and solution algorithms. Comput. Oper. Res. 55 (C), 141–152.

Akkerman, R., Farahani, P., Grunow, M., 2010. Quality, safety and sustainability in food distribution: a review of quantitative operations management approaches and challenges. Or. Spect. 32 (4), 863–904.

Andersson, H., Hoff, A., Christiansen, M., Hasle, G., Løkketangen, A., 2010. Industrial aspects and literature survey: combined inventory management and routing. Comput. Oper. Res. 37 (9), 1515–1536.

Archetti, C., Bertazzi, L., Laporte, G., Speranza, M.G., 2007. A branch-and-cut algorithm for a vendor-managed inventory-routing problem. Transp. Sci. 41 (3), 382–391.

Archetti, C., Bertazzi, L., Paletta, G., Speranza, M.G., 2011. Analysis of the maximum level policy in a production-distribution system. Comput. Oper. Res. 38 (12), 1731–1746.

Barbosa-Póvoa, A.P., Pinto, J.M., 2018. Challenges and perspectives of process systems engineering in supply chain management. Comp. Aided Chem. Eng. 44, 87–96.

Bard, J., Nananukul, N., 2010. A branch-and-price algorithm for an integrated production and inventory routing problem. Comput. Oper. Res. 37 (12), 2202–2217.

Bell, W.J., Dalberto, L.M., Fisher, M.L., Greenfield, A.J., Jaikumar, R., Kedia, P., Mack, R.G., Prutzman, P.J., 1983. Improving the distribution of industrial gases with an on-line computerized routing and scheduling optimizer. Interfaces 13 (6), 4–23.

Çetinkaya, S., Uster, H., Easwaran, G., Keskin, B.B., 2009. An integrated outbound logistics model for Frito-Lay: coordinating aggregate-level production and distribution decisions. Interfaces 39 (5), 460–475.

Chandra, P., 1993. A dynamic distribution model with warehouse and customer replenishment requirements. J. Oper. Res. Soc. 44 (7), 681–692.

Chandra, P., Fisher, M.L., 1994. Coordination of production and distribution planning. Eur. J. Oper. Res. 72, 503–517.

Cheung, K.L., Zhang, S.H., 2008. Balanced and synchronized ordering in supply chains. IIE Trans. 40 (1), 1–11.

Christiansen, M., Fagerholt, K., Nygreen, B., Ronen, D., 2007. Maritime transportation. Handb. Operat. Res. Manage. Sci. 14, 189–284.

Christiansen, M., Fagerholt, K., Nygreen, B., Ronen, D., 2013. Ship routing and scheduling in the new millennium. Eur. J. Oper. Res. 228 (3), 467–483.

Christiansen, M., Fagerholt, K., Ronen, D., 2004. Ship routing and scheduling: status and perspectives. Transp. Sci. 38 (1), 1–18.

Cóccola, M., Zamarripa, M., Méndez, C., Espuña, 2013. A toward integrated production and distribution management in multi-echelon supply chains. Comput. Chem. Eng. 57, 78–94.

Cóccola, M., Méndez, C., Dondo, R., 2017. A decomposition framework for managing inventory and distribution of fluid products by an order-based-resupply methodology. Comput. Chem. Eng. 106, 373–384.

Cóccola, M., Méndez, C., Dondo, R., 2018. Optimizing the inventorying and distribution of chemical fluids: an innovative nested column generation approach. Comput. Chem. Eng. 119, 55–69.

Coelho, L.C., Cordeau, J.F., Laporte, G., 2014. Thirty years of inventory routing. Transp. Sci. 48 (1), 1–19.

Dong, Y., Maravelias, C.T., Pinto, J.M., Sundaramoorthy, A., 2017. Solution methods for vehicle-based inventory routing problems. Comput. Chem. Eng. 101, 259–278.

Eetebari, F., Dabiri, N., 2016. A hybrid heuristic for the inventory routing problem under dynamic regional pricing. Comp. Chem. Eng. 95, 231–239.

Fumero, F., Vercellis, C., 1999. Synchronized development of production, inventory, and distribution schedules. Transp. Sci. 33, 3.

Grossmann, I., 2012. Advances in mathematical programming models for enterprise-wide optimization. Comput. Chem. Eng. 47, 2–18.

Lübbecke, M.E., Desrosiers, J., 2005. Selected topics in column generation. Op. Res. 53 (6), 1007–1023.

Marchetti, P., Gupta, V., Grossmann, I., Cook, L., Valton, P., Singh, T., 2014. Simultaneous production and distribution of industrial gas supply-chains. Comput. Chem. Eng. 69, 39–58.

Neves-Moreira, F., Almada-Lobo, B., Cordeau, J.F., Guimarães, L., Jans, R., 2019. Solving a large multi-product production-routing problem with delivery time windows. Omega 86, 154–172.

Ruokokoski, M., Solyalı, O., Cordeau, J.-F., Jans, R., Süral, H., 2010. Efficient formulations and a branch-and-cut algorithm for a production-routing problem. GERAD TechnicalReportG-2010-66. HEC, Montréal, Canada.

Savelsbergh, M., Song, J.H., 2008. An optimization algorithm for the inventory routing problem with continuous moves. Comp. Op. Res. 35 (7), 2266–2282.

Singh, T., Arbogast, J., Neagu, N., 2015. An incremental approach using local-search heuristic for inventory routing problem in industrial gases. Comp. Chem. Eng. 80, 199–210.

Solyalı, O., Süral, H., 2011. A branch-and-cut algorithm using a strong formulation and an a priori tour-based heuristic for an inventory-routing problem. Transp. Sci. 45 (3), 335–345.

Zamarripa, M., Marchetti, P., Grossmann, I., Singh, T., Lotero, I., Gopalakrishnan, A., Besancon, B., André, J., 2016. Rolling horizon approach for production−distribution coordination of industrial gases supply chains. Ind. Eng. Chem. Res. 55, 2646–2660.