OPTIMAL PLANNING AND SCHEDULING OF MULTIPRODUCT BATCH PLANTS OPERATING UNDER PRODUCTION CAMPAIGN IN A MULTIPERIOD CONTEXT

Yanina Fumero†, Gabriela Corsano†, Marta S. Moreno‡ and Jorge M. Montagna†

†Instituto de Desarrollo y Diseño, CONICET-UTN, Avellaneda 3657, S3002GJC Santa Fe, yfumero@santafe-conicet.gov.ar, gcorsano@santafe-conicet.gov.ar, mmontagna@santafe-conicet.gov.ar, www.ingar.santafe-conicet.gov.ar

‡Planta Piloto de Ingeniería Química (PLAPIQUI), CONICET-UNS, Camino La Carrindanga km 7, 8000 Bahía Blanca, Argentina, smoreno@plapiqui.edu.ar, www.plapiqui.edu.ar

Abstract: A multiperiod optimization model for the simultaneous planning and scheduling of multiproduct batch plants is presented in this work. Decisions about production and storage of different products, as well as production campaign in each period are jointly taken into account. Thus, for each time period, the number and size of batches of each product in the campaign, the assignment of batches to units and their sequencing, and the number of repetitions of the campaign must be determined. Different trade-offs among the various decision variables are evaluated according to seasonal and market fluctuations presented in each period. The proposed modelling framework provides a valuable tool for guiding the decisions in planning and scheduling of multiproduct batch plants.

Key words: *multiproduct batch plant, mixed product campaign, multiperiod MILP model*
2000 AMS Subjects Classification: 21A54 - 55P5T4

1. INTRODUCTION

Multiproduct batch plants are characterized by the production of multiple products with similar recipes. All products are produced following the same processing sequence. Batch units are characterized by a processing time and no simultaneous feed and removal is performed.

Several problems can be posed for this kind of plants. Assuming a given plant, i.e., its configuration and the unit sizes are known, different production problems can be posed depending on the requirements of the contemplated scenario. Costs, raw materials and demands typically vary from period to period due to market or seasonal reasons, therefore several time periods must be considered. In this context, scheduling is a critical decision for the operational efficiency of the plant. In particular, many approaches have been presented for the scheduling of batch processes. However, they have been mainly focused on short term scheduling ([1], [2]), where a set of production orders must be processed using the available resources.

From the planning perspective, several approaches have assumed that the plant operates using single product campaigns (SPCs) in each period ([3]) in order to reduce the problem formulation and simplify its solution. In this policy, all the batches of a product are produced without overlapping with other products. Although multiperiod approaches allow obtaining more flexible production programs, SPCs are not suitable because they can overestimate the time requirements, lead to inventory buildups of materials, and may be impracticable when perishable products are considered.

Mixed product campaigns (MPCs) can be adopted to overcome these drawbacks and improve productivity of multiproduct batch facilities. Fumero et al. [4] have addressed planning and scheduling decisions based on this type of campaigns for plants operating in a regular fashion along a time horizon. They showed that this policy provides a more steady supply of products, reducing idle times and increasing the utilization of units. In spite of these advantages, the incorporation of constraints for MPCs requires a more complex formulation ([5]).

Considering this context, this work presents a mixed integer linear programming model (MILP) that simultaneously addresses multiproduct batch plants planning and scheduling decisions over different time periods with an operation based on campaigns. Deterministic variations in prices, product demands limits, costs, and raw materials availability due to seasonal or market fluctuations are included in this approach. Unlike previous presented models ([3], [4]), this approach considers MPC in a multiperiod scenario. The proposed formulation is posed as an appropriate tool for decision making in supporting the plant management.

2. PROBLEM STATEMENT

A multiproduct batch plant that processes I products during T time periods of duration H_t is proposed. All products follow the same production sequence throughout J batch processing stages of the plant and they are produced using C raw materials. Some stages have units of the same size duplicated out of phase. No intermediate storage tanks are allocated between stages and the Zero Wait (ZW) transfer policy is adopted. This policy assumes that a batch, after finishing its processing at a stage, must be transferred immediately to the next stage.

Production of product i at stage j in every period t requires a given processing time t_{ijt} and a material balance factor S_{ijt} called the size factor, which specifies the volume required at stage j to produce a unit mass of final product i. For each product i, lower and upper bounds on its demands in every period t, DE_{it}^{L} and DE_{it}^{U}, are known. The amounts of raw materials consumed are determined by mass balances with a given parameter F_{cit} that accounts for the process conversion of raw material c to produce product i during period t. Costs and availability of raw materials vary from period to period and are assumed to be known. Also, prices of final products in each time period, and maximum available storage capacities, are problem data. At the beginning of the time horizon, the initial inventories of both raw material and product, IM_{i0} and IP_{i0}, are assumed to be given.

During each time period, the plant operates in MPC mode, i.e. the production campaign, composed by a set of batches of the different products manufactured in this period, is cyclically repeated over H_t. For each product, a maximum number of batches in the campaign of period t, NBC_{it}^{U}, is given. An asynchronous slot-based continuous-time representation for modeling the assignment of batches to units is employed [5].

Then, the problem consists of determining for each time period t: (1) the amounts of product i to be produced in period t, q_{it}, raw material c to be used for production in period t, RM_{ct}, purchased raw material c during period t, C_{ct}, and wasted raw material, RW_{ct}; (2) the number and size of the batches for each final product i elaborated in period t; (3) the MPC composition, i.e. the number of batches of each product in the campaign; (4) the assignment of batches to units in each stage, production sequence on each unit, initial and final processing times for the batches in each unit and the campaign cycle time, CTC_t; (5) the number of times that the campaign is cyclically repeated over the time horizon H_t, denoted by NN_t; (6) the levels of both final product, IP_{it}, and raw material inventories, IM_{it}; (7) the wastes due to the expired product shelf life, PW_{it}; and (8) the total sales of each product i elaborated, QS_{it}.

The performance criteria adopted is the net benefit maximization.

3. MODEL FORMULATION

The model basically considers: i) production planning constraints, that allow determining, at each time period, the amount of raw materials purchased and used for producing each product, the total production, the levels of raw material and final product inventories, and the amount of sold products, and, ii) scheduling constraints which assign batches to specific slots at each unit, timing constraints for determining the initial and final times of processed slots, equations for representing the ZW policy, and the campaign cycle time calculations for each time period. Several constraint reformulations and additional binary variables are introduced to keep the problem linear. Finally, the objective function maximizes the benefit over the time horizon. This economic criterion is calculated by the difference between the revenue due to product sales and the overall costs, with the latter consisting of the purchases of raw materials, inventories, operation, and late delivery penalties costs.

Due to space reasons, the detailed formulation is not provided in this manuscript, but readers can request it to the authors.

4. EXAMPLE

The production planning and scheduling of a given multiproduct batch plant that involves four processing stages with two identical units operating out-of-phase on stage 1, are solved. Unit sizes in each stage are 4000, 2500, 1500 and 3000 L, respectively. The plant produces 3 products with 2 different raw materials. A planning horizon of 1000 h with 4 equal time periods of 2 weeks (250 h) is considered.

Prices of raw materials and final products, and maximum bounds on demand forecasts over each period, are given in Table 1. Minimum product demands in each period are assumed as 50% of maximum product demands. Due to space reasons, data on processing times, size and conversion factors, products and raw materials lifetimes in time periods, and inventory costs of both final products and raw materials was not reported. However, they are available for everyone who requests them.

For each period, the number of batches of product *i* in the composition of the production campaign is upper bounded by $NBC_{it}^{U} = 3$.

Table 1. Prices and demand bounds

	Raw material costs ($/kg)		Products prices ($/kg)			Maximum demands (× 10^3 kg)		
t	C1	C2	I1	I2	I3	I1	I2	I3
1	1.0	0.5	2.05	2.60	2.00	12.0	10.5	9.5
2	1.5	0.8	2.25	2.60	2.20	12.6	10.9	10.0
3	1.5	0.5	2.25	2.40	2.20	13.2	11.5	10.5
4	1.0	0.8	2.05	2.40	2.00	13.8	12.0	10.9

The lower and upper bounds for the variable representing the number of repetitions of the campaign over H_t, NN_t, are proposed considering the two extreme types of campaigns that can be arisen in each time period, i.e. that with minimum cycle time and that with maximum cycle time. For this example, NN_t is uniformly discretized, considering a step size equal to 2, over the interval [6, 24]. Then the recurrence relation $T_{mt} = T_{m\text{-}1t} + 2$ for $2 \leq m \leq 10$ with $T_{1t} = 6$, allows defining the discrete multiple choice for NN_t.

The model under these assumptions comprises 13021 linear constraints, 3053 continuous variables, and 484 binary variables. It was implemented and solved using GAMS, via CPLEX 12.1 solver, in 89.95 CPU seconds with a 0% of optimality gap. The optimal solution has a value of $98921.

For each period, the amounts of final products produced and sold, amounts of raw materials purchased for producing all products, and the inventories levels of both raw materials and products, are summarized in Table 2.

Table 2. Optimal production plan for each time period

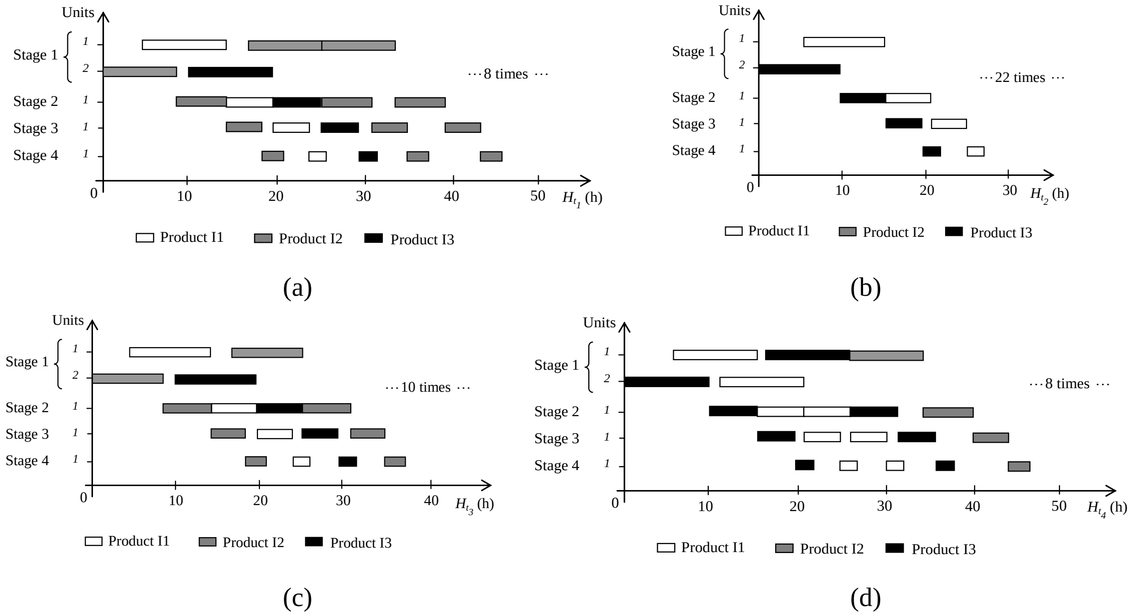
	I1 (× 10^3kg)			I2 (× 10^3kg)			I3 (× 10^3kg)			C1(×10^3kg)		C2(×10^3kg)	
t	q_{it}	QS_{it}	IP_{it}	q_{it}	QS_{it}	IP_{it}	q_{it}	QS_{it}	IP_{it}	C_{ct}	IM_{ct}	C_{ct}	IM_{ct}
1	6.4	6.2	0.2	20.4	10.5	9.9	5.0	4.8	0.2	70.6	43.5	79.3	40.2
2	17.6	12.6	5.2	0.0	9.9	0.0	13.8	10.0	4.0	0.0	25.1	0.0	0.0
3	8.0	13.2	0.0	16.7	11.5	5.2	6.2	10.2	0.0	0.0	0.0	75.7	37.3
4	12.8	12.8	0.0	6.8	12.0	0.0	10.0	10.0	0.0	20.2	0.0	0.0	0.0

Both raw materials are purchased in periods where costs are the lowest ones. For raw material C1, the extra material purchased in period 1 is kept as inventory for fulfilling production in the next two periods. Analogously, for raw material C2, the extra material purchased in periods 1 and 3 are kept as inventory for production in subsequent periods. For each product, extra amounts are produced in some periods, which are kept as inventory to satisfy maximum demands in subsequent periods. For each period, the composition of optimal production campaign, its cycle time and the number of times that it is repeated over the time horizon are depicted in Table 3. Figure 1 illustrates the production sequence in the different stages for all time periods.

Table 3. Optimal production campaign for each time period

	NBC_{it}			CTC_t (h)	NN_t
t	I1	I2	I3		
1	1	3	1	31.0	8
2	1	0	1	10.9	22
3	1	2	1	22.5	10
4	2	1	2	30.6	8

Since product I2 is not produced in period 2, the campaign only consists of one batch of each of the other products, which is sufficient to meet the production plan in this period.



(a) (b) (c) (d)

Figure 1. Gantt chart of the optimal MPC for periods: (a) 1, (b) 2, (c) 3 and (d) 4.

5. CONCLUSIONS

A multiperiod MILP model for the simultaneous planning and scheduling of a given multistage multiproduct batch plant with parallel units, that operates with a campaign-based mode, was presented.

The proposed model simultaneously determines for each time period the resources use, product and raw material storages, the number and size of batches for each product, the composition of the production campaign, the assignment of batches to units, the production sequence on each unit, initial and final processing times for the batches processed in each processing unit, and the number of times that the campaign is cyclically repeated, fulfilling the product demand limits over the specified global time horizon.

The operation management and production planning are common activities that are approached in the plant floor, due to seasonal changes, fluctuation in product demands, or market variation, among others. The proposed model allows making different decisions, like the forecast of material requirement, the inventory management of raw materials and final products, the distribution policy determination, etc. Therefore, the plant can be appropriately operated and controlled using this model.

6. REFERENCES

[1] C. A. Méndez, J. Cerdá, I. E. Grossmann, I. Harjunkoski, M. Fahl, *State-of-the-art review of optimization methods for short-term scheduling of batch processes*, Comput. Chem. Eng., 30 (2006), pp. 913-946.

[2] M. Pan, X. Li, and Y. Qian. *Continuous-time approaches for short-term scheduling of network batch processes: Small-scale and medium-scale problems*, Chem. Eng. Res. Des. 2009, 87, 1037-1058.

[3] M. Moreno, J. M. Montagna, and O. Iribarren. *Multiperiod optimization for the design and planning of multiproduct batch plants*, Comput. Chem. Eng., 31 (2007), pp. 1159-1173.

[4] Y. Fumero, G. Corsano, and J. M. Montagna. *Planning and Scheduling of multistage multiproduct batch plants operating under production campaigns*, Ann. Oper. Res., 199 (2012), pp. 249-268.

[5] Y. Fumero, G. Corsano, and J. M. Montagna. *Scheduling of multistage multiproduct batch plants operating in a campaign-mode*, Ind. Eng. Chem. Res., 51 (2012), pp. 3988-4001.