Dynamic transition in conveyor belt driven granular flow

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Abstract

We consider the flow of disks of diameter d driven by a conveyor belt of dynamic friction coefficient μ through an aperture on a flat barrier. The flow rate presents two distinct regimes. At low belt velocities v the flow rate is proportional to v and to the aperture width A. However, beyond a critical velocity, the flow rate becomes independent of v and proportional to $(A - kd)^{3/2}$ in correspondence with a two-dimensional Beverloo scaling. In this high-velocity regime we also show that the flow rate is proportional to $\mu^{1/2}$. We discuss that these contrasting behaviors arise from the competition between two characteristic time scales: the typical time a disk takes to stop on the belt after detaching from the granular pack and the time it takes to reach the aperture.

Keywords: Conveyor belt, Granular flow, Granular discharge

1. Introduction

Conveyor belts are widely used in the industry to transport granular matter in bulk. From raw materials, to seeds to cans and bottles, conveyor belts are chief in moving materials minimizing relative granular flow [1]. When a constriction is used to force grains to flow through a narrower section, the flow rate has to accommodate. The phenomenon is similar to the one observed during the discharge of a vertical silo through an opening in the base. Despite the many studies carried out in silo discharge (see for example [2, 3] and references therein), little has been discussed on the belt driven flow rate trough a bottleneck.

De Song et al. [4] have shown that a critical transition exists for a given belt velocity v_c. These authors carried out experiments of the discharge of disks of diameter d that are driven by a belt at constant v towards a barrier with an aperture of width A. For velocities below v_c they show that the flow rate is proportional to the belt velocity. Beyond v_c the flow rate seems to presents a new linear dependence with v showing a lower slope.

More recently, Aguirre et al. have argued that for $v > v_c$ the flow rate should become independent of v [5, 6]. Moreover, they put forward a simple argument for the existence of the transition and infer an expression for v_c as a function of the belt–disk dynamic friction coefficient μ and the aperture width A.

In this work, we carry out discrete element method (DEM) simulations of disks on a conveyor belt flowing pass an aperture on a flat barrier for a wide range of belt velocities. We show that the transition described by De Song et al. is in accordance with the predictions of Aguirre et al. However, the experiments in Ref. [4] are conducted at intermediate velocities and were unable to show the full range of responses predicted.

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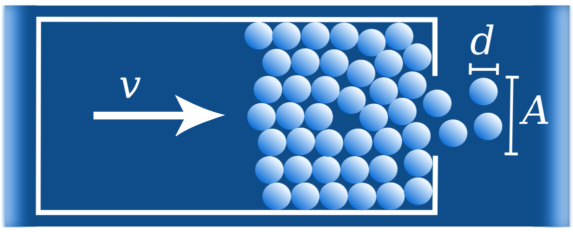

Figure 1: Schematic diagram of the belt conveying disks of diameter d through an aperture of width A at velocity v.

The second linear regime described in Ref. [4] is only a narrow portion of a transition regime between the low-velocity and the high-velocity regimes of the phenomenon. A simple criterion allowed us to define v_c in terms of the relative velocity of the disks with respect to the belt at the time they exit. We will present results for different μ and A and show that the predicted dependence of v_c on these variables are indeed observed in our simulations.

The rest of the paper is organized as follows. In section 2, we summarize the theoretical predictions. In section 3, the details of the DEM simulations are given. Section 4 presents the results of the simulations and a discussion in view of previous studies. The conclusion are drawn in section 5.

2. Flow regimes in a conveyor belt driven discharge

Consider a set of disks of diameter d and mass m on a conveyor belt with disk–belt dynamic friction coefficient μ. The disks are contained by a rectangular frame with an aperture on the "bottom" side towards they are dragged to at constant velocity v (see Fig. 1). Disks will pack against this side and flow pass the orifice. When a steady flow is set, inside the confining frame disks in the pack rearrange while the conveyor belt slides beneath until they set free from the the packing (loosing contact with neighbors) and move away through the opening.

Each disk that sets free requires a period of time t in order to stop on the conveyor belt and move with it at v. During this time, the disk accelerates with constant acceleration $a = \mu g$ (each disk of mass m is subjected to the dynamic fiction force $mg\mu$). Therefore, to change its velocity in the laboratory reference system from zero to the belt velocity v, the disk needs a lapse of time $t = v/(\mu g)$. Over this time the disk travels a distance

$$x = \frac{a}{2}t^2 = \frac{v^2}{2\mu g}. \quad (1)$$

If x is small (low-velocity regime), disks that detach from the packing will attain the belt velocity v before reaching the aperture. Since the flow rate Q is proportional to the velocity of the outflowing disks, $Q \propto A_{eff}v$. Here A_{eff} is an effective outlet width that accounts for the boundary effects at the edges of the aperture (the so called *empty annulus* [7]). As it is customary, we take $A_{eff} = A - kd$, and select k to fit the data. In the discharge of three-dimensional silos $k \approx 1.0$ [2, 3], however, in two-dimensional setups $k \approx 3.0$ [3]. It is worth mentioning that this empty annulus effect can be taken into account in a much careful fashion by considering the velocity profile across the aperture width as done by Janda et al. [8]. We will follow the simpler traditional correction since we are focusing here on a different aspect of the flow rate.

If the acceleration phase takes some time, disks will reach the outlet still with acceleration a (high-velocity regime). In this case, the characteristic velocity of the outflowing disks can be estimated as $\sqrt{aA_{eff}/2}$. Here, we have made the rough assumption that disks detach from the rest of the pack at a distance $A_{eff}/2$ before reaching the outlet. Then, in the high-velocity regime,

$$Q \propto A_{eff}\sqrt{aA_{eff}} = \sqrt{\mu g}A_{eff}^{3/2}. \quad (2)$$

This corresponds to the two-dimensional Beverloo scaling [8]. In the high-velocity regime, we should therefore observe a discharge of accelerated grains similar to the one observed in silos with an effective acceleration μg. It is important to notice that the flow rate becomes independent of the belt velocity v in this regime. Therefore, one should expect no improvement in flow rate by increasing v at high velocities.

We can estimate the critical belt velocity v_c around which a change of regime (low- to high-velocity) is expected. If the distance x traveled by disks before stopping on the belt is greater than the distance from the point of detachment of the grains from the rest of the packing to the plane of the orifice, which we approximate as $A/2$, most disks will exit still accelerated. Hence, from Eq. (1) we can predict

$$v_c = C_c\sqrt{g\mu A}, \quad (3)$$

with C_c a proportionality constant. Notice that v_c is independent of the size and mass of the disks, only A and μ can be used to tune v_c. Equations (2) and (3) were put forward by Aguirre et al. [5] without empirical evidences.

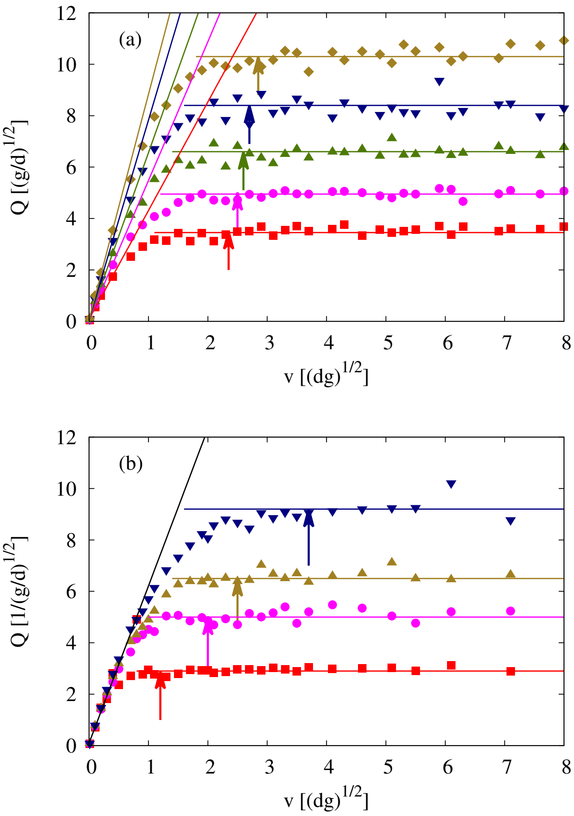

Figure 2: (a) Flow rate Q as a function of belt velocity v for $\mu = 0.5$ and various outlet widths $A = 6.0d$ (red squares), $7.0d$ (magenta circles), $8.0d$ (green up triangles), $9.0d$ (blue down triangles), $10.0d$ (yelow diamonds). (b) Same as part (a) for $A = 8.0d$ and various belt–disk friction coefficients $\mu = 0.1$ (red squares), 0.3 (magenta circles), 0.5 (yellow up triangles), 1.0 (blue down triangles) The straight lines correspond to linear fits to the data at low and high v. The arrows indicate the critical velocity v_c defined in terms of the probability of finding disks flowing out accelerated (see main text and Fig. 4).

In the rest of the paper we will show that these predictions are confirmed in numerical simulations of the conveyor belt. We will use a simple criterion to obtain v_c from the simulations. Moreover, we will show that the transition region between the two extreme regimes, which is of interest in many industrial applications, is rather wide.

3. DEM simulation

We have followed the standard techniques on discrete element methods (see for example Refs. [9, 10]). We used a velocity Verlet algorithm to integrate the Newton equations for N monosized disks (diameter d and mass m) in a rectangular box of width $L = 25d$. We studied system sizes between $N = 1000$ and $N = 2000$.

The disk–disk and disk–wall contact interaction comprises a linear spring–dashpot in the normal direction

$$F_n = k_n\xi - \gamma_n v^n_{i,j} \quad (4)$$

and a tangential friction force

$$F_t = -\min(\mu|F_n|, |F_s|)\cdot \mathrm{sign}(\zeta) \quad (5)$$

that implements the Coulomb criterion to switch between dynamic and static friction [11].

In Eqs. (4)–(5), $\xi = d - \left|\mathbf{r}_{ij}\right|$ is the particle–particle overlap, $\mathbf{r}_{ij}$ represents the center-to-center vector between particles i and j, $v^{\mathrm{n}}_{i,j}$ is the relative normal velocity, $F_{\mathrm{s}} = -k_{\mathrm{s}}\zeta - \gamma_{\mathrm{s}} v^{\mathrm{t}}_{i,j}$ is the static friction force, $\zeta(t) = \int_{t_0}^{t} v^{\mathrm{t}}_{i,j}(t')\,dt'$ is the relative shear displacement, $v^{\mathrm{t}}_{i,j} = \dot{\mathbf{r}}_{ij} \cdot \mathbf{s} + \frac{1}{2} d\left(\omega_i + \omega_j\right)$ is the relative tangential velocity, and $\mathbf{s}$ is a unit vector normal to $\mathbf{r}_{ij}$. The shear displacement ζ is calculated by integrating $v^{\mathrm{t}}_{i,j}$ from the beginning of the contact (i.e., $t = t_0$). The disk–wall interaction is calculated considering the wall as an infinite radius, infinite mass disk. The interaction parameters are the same as for the disk-disk interaction.

In these simulation we used the following set of parameters: friction coefficient $\mu = 0.5$ (notice that in this type of simulations $\mu_{\mathrm{dynamic}} = \mu_{\mathrm{static}}$), normal spring stiffness $k_n = 10^5(mg/d)$, normal viscous damping $\gamma_n = 300(m\sqrt{g/d})$, tangential spring stiffness $k_s = \frac{2}{7}k_n$, and tangential viscous damping $\gamma_s = 200(m\sqrt{g/d})$. The integration time step is $\delta = 10^{-4}\sqrt{d/g}$. Units are reduced with the diameter of the disks, d, the disk mass, m, and the acceleration of gravity, g.

Disks lay flat on a belt that moves at constant velocity v. The belt–disk interaction is modeled only via the tangential force in Eq. (5). In this case, we have used different friction coefficients μ for the belt-disk interaction. Every disk is considered to be always in contact with the belt exerting a normal force $F_{\mathrm{n}} = mg$. The frictional force due to the rotation of disks is not taken into account in our simulations. We assume that this force has very little impact on the dynamics.

Disks initially placed at random without overlaps on the belt are dragged towards "the base" of the confining box. After all disks are piled against the base, we open an aperture of width A in the base and record the flow of disks through it. The initial transient flow is disregarded in our analysis, only the steady state flow will be reported. For some set of parameters we have repeat some of the simulations up to 10 times with different initial conditions to estimate the error in the flow rate. Since we have found these errors are rather small, we report in all cases the flow rate obtained from a single discharge for each set of A, μ and v.

4. Results

In Fig. 2, we plot the flow rate, in number of disks per unit time, as a function of the belt velocity for various aperture sizes A and belt–disk friction coefficients μ. As we have discussed in section 2, the flow rate presents a linear increase at low v and saturates to a constant value for high v. The transition region between these two extreme regimes is somewhat wide. While the full development of the high-velocity regime occurs after v_c surpasses 3 to 4 $\sqrt{dg}$ (depending on A and μ), the low-velocity linear increase is departed from soon after $v > 0.1v_c$.

In a previous study by de Song et al. [4], the maximum belt velocities attained where roughly $3.0\sqrt{dg}$. Only for the smaller apertures this maximum velocity is just into the high-velocity regime. This prevented the clear observation of the plateau we report in Fig. 2, which was later anticipated by Aguirre et al. [5].

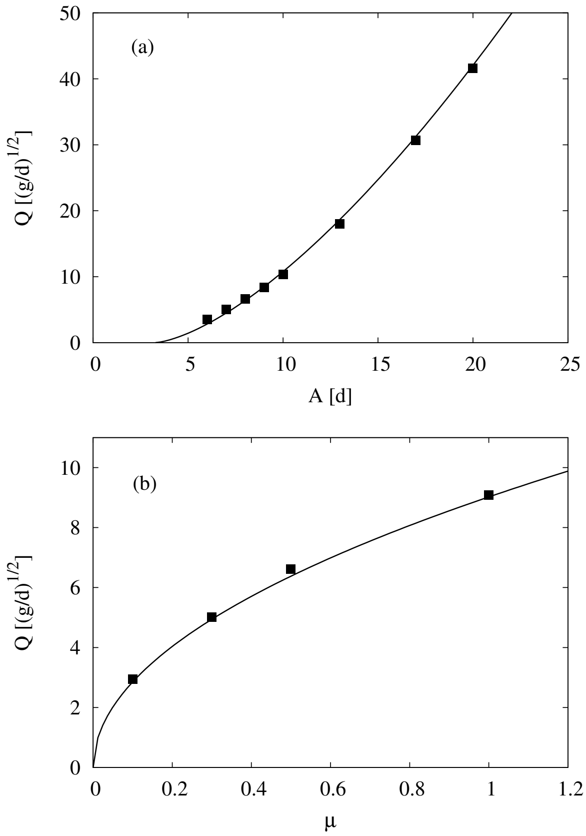

Figure 3: (a) Flow rate Q in the high-velocity regime as a function of the width A of the aperture for $\mu = 0.5$. (b) Same as (a) as a function of μ for $A = 8.0d$. The solid line corresponds to the Beverloo rule ($Q = C\sqrt{\mu g}(A - kd)^{3/2}$) with $C = 0.866$ and $k = 3.23$.

The saturation value for Q at high v depends both on A and μ as predicted by Eq. (2) (see Fig. 2). However, the slope of the low-v regime depends only on A. Indeed, as we mentioned in section 2, at low velocities all disks pass the outlet at the belt velocity regardless of μ. In Fig. 3, we show the saturation flow rate as a function of A and μ. The results are in good agreement with Eq. (2), displaying a 3/2-power dependence of Q with A and a 1/2-power dependence with μ. This confirms that the flow in the high-velocity regime is analogous to a gravity driven discharge of a two-dimensional silo with an effective gravity given by μg. Notice that $A_{eff} = A - kd$ and the fit yields $k \approx 3$ in accordance with Ref. [3].

In order to define a critical velocity beyond which the flow becomes independent of v, we have measure the component of the velocity of the disks in the direction of the belt velocity when they reach the opening. Disks that have attained the belt velocity are at rest with respect to the conveyor belt, disks that have a lower velocities are still sliding on the belt. We define the probability P of a disk exiting at the belt velocity v as the fraction of disks that reached v when their centers cross the plane of the aperture. In Fig. 4, we show this probability as a function of v for different A and μ. At low v, P shows that about 80%

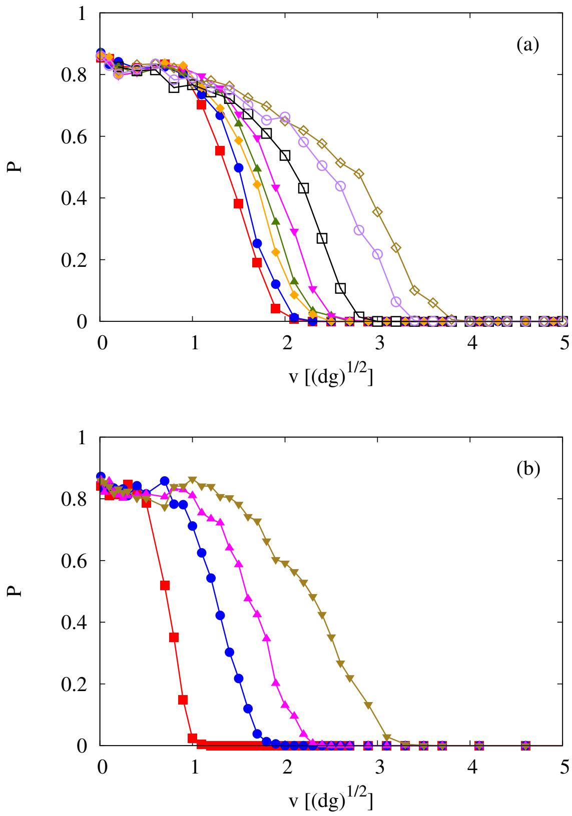

Figure 4: (a) Probability of finding a disk leaving the confining frame at the belt velocity as a function of the belt velocity v for $\mu = 0.5$ and $A = 6.0d$ (red filled squares), $7.0d$ (blue filled circles), $8.0d$ (orange filled diamonds), $9.0d$ (green up triangle), $10.0d$ (magenta down triangle), $13.0d$ (black hollow squares), $17.0d$ (purple hollow circles), $20.0d$ (yellow hollow diamonds). (b) Same as (a) for $A = 8.0d$ and μ=0.1 (red squares), 0.3 (blue circles), 0.5 (magenta up triangles) and 1.0 (yellow down triangles).

of the disks exit at velocity v. Although we may have expected that all disk reach the belt velocity at very low v, we have to consider that disks at the edges of the orifice interact with the "bottom" barrier and their velocities are in general smaller than v. Interestingly, P presents this 80% value for all apertures studied, indicating that not only the disks touching the edges of the aperture are affected in their velocity but also other disks at the exit line.

From Fig. 4, we can observe that P falls to zero at a well defined value of v for each A and for each μ. Beyond this zero-P point, all disks leave the confining frame at a velocity lower than the belt velocity, meaning that they are still accelerating when crossing the exit line. We use this as a definition of the critical velocity v_c for given A and μ. The critical velocities so obtained are indicated with arrows in Fig. 2.

The critical velocity obtained from Fig. 4 is plotted as a function of A and μ in Fig. 5. As we can see, the square root dependency on A and μ predicted by Eq. (3) is confirmed in the simulations. Hence, Eq. (3) allows us to estimate the maximum belt velocity for which an improvement in throughput can be expected from an increase of v.

The predicted v_c sets the range where Q can be predicted

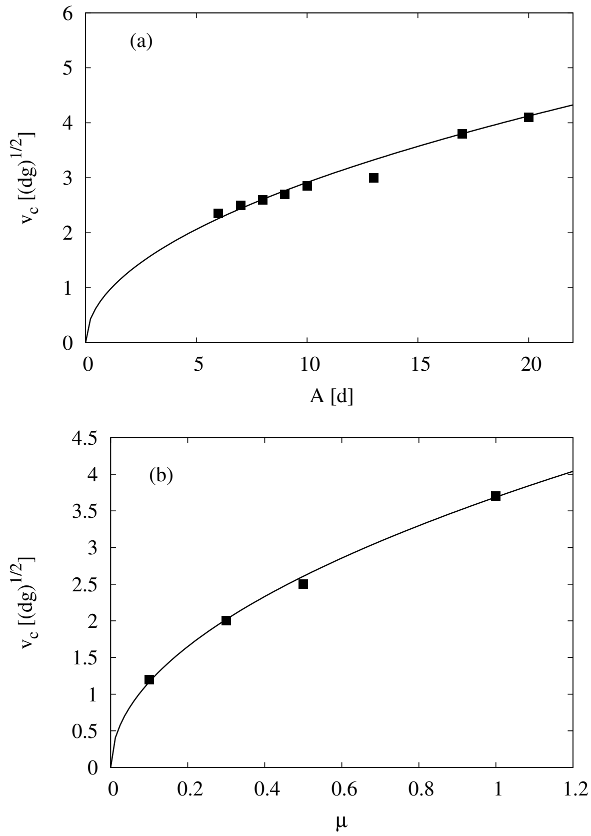

Figure 5: (a) Critical velocity v_c as a function of the aperture width A for $\mu = 0.5$. (b) Same as part (a) as a function of μ for $A = 8.0d$. The values have been obtained as the belt velocity for which all disks leave the confining frame with velocities below the belt velocity (see Fig. 4). The lines in the plots correspond to fits using Eq. (3) with $C_c = 0.92$.

with simple arguments. For $v > v_c$, Eq. (3) holds, for $v < 0.1v_c$ $Q = A_{eff}v$. For $0.1v_c < v < v_c$, the mixed dynamics with disks exiting at the belt velocity in coexistence with disks exiting accelerated gives rise to a flow rate non-trivially dependent of v. Interestingly, this intermediate regime may be of particular interest to many industrial conveyor belts.

5. Conclusions

We have shown by means of DEM simulations that there exist two extreme regimes in the flow rate of disks driven by a conveyor belt through an aperture in a barrier. At low velocities the flow rate is proportional to the belt velocity, whereas at high velocities the flow rate reaches a plateau. This implies that at high belt velocities there is no improvement in throughput.

In the high velocity regime, the flow rate follows the two-dimensional version of the Beverloo rule with a 3/2-power dependency on the aperture width and an effective gravity given by μg. The maximum attainable flow rate can therefore be controlled only through the aperture size or the belt–disk friction coefficient. Equation (2) provides an expression to calculate the maximum flow rate.

Our results indicate that previous studies [4] where a dynamic transition was observed, explored a limited range of belt velocities, which prevented the full development of the high velocity regime.

The transition region of intermediate velocities has been found to be rather wide. Many industrial conveyor belts may work at these velocities. Unfortunately, at present, a model to predict the flow rate at these intermediate velocities is still laking.

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